Economics of Governance (2007) 8:197–218
DOI 10.1007/s10101-006-0026-8

ORIGINAL PAPER

A theory of rent seeking with informational foundations

Johan N. M. Lagerlöf

Received: 19 September 2005 / Accepted: 27 October 2006 / Published online: 29 November 2006

Abstract I develop a model of rent seeking with informational foundations and an arbitrary number of rent seekers, and I compare the results with Tullock's (1980) classic model where the influence activities are "black-boxed." Given the microfoundations, the welfare consequences of rent seeking can be studied. In particular, I show that competition among rent seekers can be socially beneficial, since the additional information that the decision maker gets access to makes the increase in rent-seeking expenditures worthwhile. However, the analysis also highlights a logic that, under natural parameter assumptions, makes the rent seekers spend more resources on rent seeking than is in society's interest, which is consistent with the spirit of the rent-seeking literature.

Keywords Rent seeking · Competition · Lobbying · Information acquisition · Disclosure · Welfare

JEL Classification D42 · D43 · D72 · D83 · L13

1 Introduction

In a very influential paper, Tullock (1980) introduced a model of rent seeking in which n economic agents try to win an indivisible prize, for example, the

J. N. M. Lagerlöf (✉)
Economics Department, Royal Holloway,
University of London,
Egham TW20 0EX, Surrey, UK
e-mail: johan.lagerlof@rhul.ac.uk

J. N. M. Lagerlöf
CEPR, London, UK

profits associated with being granted a monopoly of a market.[1] They do so by investing resources into an (abstract, not explicitly modeled) rent-seeking activity: the more an agent invests, the greater the likelihood that it wins the prize. In particular, Tullock assumed that each agent's probability of winning is given by an exogenous function, parameterized by a constant R. This constant is a measure of how discriminatory the probability-of-winning function is: in the limit as R approaches infinity, a rent seeker who spends (perhaps just slightly) more than his rivals wins with probability one; similarly, in the limit as R approaches zero, each rent seeker wins with probability $1/n$, regardless of the relative magnitude of the investment levels. Tullock also assumed that the rent seekers are identical, that they have a common valuation of the prize, and that they simultaneously choose their investment levels with the objective of maximizing their expected net gain.

In the Nash equilibrium of Tullock's model, the sum of all agents' rent-seeking expenditures equals a certain fraction of the value of the prize. As the number of firms goes to infinity, this fraction approaches R.[2] Therefore, when competition among the rent seekers is very stiff, the parameter R tells us what share of the prize (e.g., the monopoly rent) that is dissipated. An important part of the rent-seeking literature has been concerned with the question whether, or to what extent, the rents (of for example a monopoly) will be dissipated by socially wasteful expenditures to capture them; see for example Nitzan (1994) and Mueller (2003, Chap. 15).

Tullock's model, being a reduced form, begs the question exactly why the rent seekers are influential. Similarly, it is not clear from Tullock's formulation why the rent-seeking expenditures are wasted from a social welfare point of view – it is just assumed that they are. One particular consequence of this assumption is that, in Tullock's model, competition among rent seekers is always bad, as a larger number of rent seekers leads to more total rent-seeking expenditures.

Very early, Tullock (1975) suggested himself that one can interpret the rent-seeking expenditures as costs of information gathering, and the reason why the rent seekers are influential could be that they (strategically) provide the decision maker with this policy-relevant information.[3] Tullock's own leading example was a trial. Another example could be the competitive bidding to host the Olympic games. Prior to the vote among the members of the International Olympic Committee, prospective host cities provide the delegates with information concerning their ability to host the games. Presumably, this information

[1] The idea of rent seeking was first discussed by Tullock (1967), although the term itself was first introduced by Krueger (1974). A third early and seminal contribution is Posner (1975). For surveys of the rent-seeking literature, see Nitzan (1994) and Mueller (2003, Chap. 15).

[2] The existence of a pure-strategy equilibrium requires that $R \leq 1$.

[3] In his classic industrial organization textbook, Tirole (1988, Chap. 1, f.n.26) also argued that informational asymmetries should be a natural way of providing foundations to the rent-seeking argument: "The analysis here is very vague. What is needed is an equilibrium model in which lobbying activities have influence. Incomplete information ought to be the key to building such a model that would explain why lobbying occurs (information, collusion with decision makers, and so on) and whether lobbying expenses are socially wasteful."

does – at least partially – concern issues that are of great importance for the question which city is most likely to deliver a successful Olympics. For example, the information could concern the prospective host city's ability to take care of local transportation and to deal with security issues during the games.[4]

Also examples that are closer to the most common ones invoked in the rent-seeking literature – like a situation in which a regulator chooses which firm to grant a natural monopoly – may fit the informational story. Prior to the regulator's decision, prospective monopolists offer information to the regulator, which could concern questions such as the efficiency of the prospective monopolist's production process, or to what extent the preferences of the consumers are well served by the features of this particular firm's good or service.

Although the idea of rent seeking as strategic information gathering and transmission is plausible, it is not clear whether such a story is properly captured by Tullock's reduced form. For if the information that is acquired is relevant for the decision which rent seeker ought to (from a social welfare point of view) be granted the prize, then the expenditures are not necessarily wasted. Moreover, we would not in general expect competition among rent seekers to be bad.

In this paper I present a relatively simple model of competitive rent seeking with informational foundations. I explicitly model how n prospective monopoly firms gather (hard, i.e., verifiable) information concerning their efficiency and then strategically choose whether to disclose the information to a regulator. In light of this information, the regulator chooses which firm to grant the monopoly. This way of thinking about rent seeking may look unreasonably benevolent, and one might be worried that it biases the results in favor of a view that rent seeking is after all not that harmful or wasteful. In spite of this possible bias, however, the analysis highlights a logic that, consistent with the spirit of the rent-seeking literature, tends to make the firms spend more resources on information gathering than is in society's interest.

To see this, note that if society could give the firms instructions as to how much information to gather, the firms would behave as if the prize that entered their objective functions was the *difference* in social welfare between having an efficient firm and having an inefficient firm – for it is this difference that society can gain by, thanks to the additional information, being able to grant the monopoly to an efficient firm rather than an inefficient one. In contrast, the prize that actually enters the firms' objective functions is the monopoly profits *conditional on being efficient*, the reason being that this is the profit they can earn by successfully gathering information showing that they are efficient. Thus, even if society's and a firm's valuations of that firm's being granted the monopoly were identical, both for the case where the firm is efficient and inefficient, the firm's perceived prize would be greater as it cares about the profits

4 Of course, the Olympic games is also a good example of a rent-seeking process that is likely to involve other kinds of influence activites, such as the offering of bribes. We know for sure that this happened in the process leading up the decision to award Salt Lake City the 2002 winter Olympics; see, for example, "Timeline: Olympics corruption scandal" at the BBC website <http://news.bbc.co.uk/1/hi/world/297030.stm> (accessed August 22, 2006).

conditional on being efficient whereas society would care about this profit/welfare level *less* the one where the firm is inefficient.

The analysis also shows that in the limit, as the number of firms approaches infinity, the total expected amount of rent-seeking expenditures equal a constant (which can take values between zero and unity) times the perceived prize, which as explained above equals the monopoly profits conditional on being efficient. Thus, this constant is a direct analogue to the parameter R in Tullock's model. In the model with informational foundations, however, the constant can be related to fundamentals of the model: it is equal to the inverse of the elasticity of the cost function associated with information acquisition, evaluated at zero (for when the number of firms is infinite, each one of them acquires a zero amount of information).

Finally and most importantly, the microfoundations of the model allow us to conduct a meaningful welfare analysis. As already mentioned, the equilibrium of Tullock's model has the property that total rent-seeking expenditures increase as the number of rent seekers increases; moreover, since rent-seeking expenditures are (by assumption) wasted, welfare moves in the opposite direction to total rent-seeking expenditures. In the present model with informational foundations, I derive a condition that determines whether expected welfare and total rent-seeking expenditures move in opposite directions or in the same direction as the number of rent seekers increases.[5] The parameters entering this condition are (i) the monopoly profits for an efficient firm, (ii) the difference in social welfare between having an efficient monopoly firm and having an inefficient monopoly firm, (iii) the inverse of the elasticity of the cost function associated with information acquisition, and (iv) the prior probability that a firm is efficient. For many configurations of these parameters, expected welfare increases as the number of rent seekers becomes larger – competition among rent seekers is welfare enhancing.[6]

The present paper is by no means the first one to develop a model of influence activities with informational foundations. Formal game-theoretic models in which lobbying is conceived as an exercise in strategic information transmission can be found in, for example, Austen-Smith and Wright (1992), Potters and van Winden (1992), Austen-Smith (1995), Lagerlöf (1997), Grossman and Helpman (2001), Bennedsen and Feldmann (2002), and Frisell and Lagerlöf (2006). Unlike the present paper and Tullock (1980), however, these contributions allow only for either one or two lobbyists. Therefore they are not able to study the effects of a gradual increase in the number of lobbyists/rent seekers on for example welfare, nor can they say anything about the degree of rent dissipation in the limit as the number of rent seekers goes to infinity.

5 Although it remains to be proven analytically, total rent-seeking expenditures appear to be increasing in the number of firms for all parameter values of the model, just as in the Tullock model (see the examples in Sect. 6).

6 Becker (1983) argues that competition between interest groups is welfare enhancing. Just like in Tullock (1980), however, the influence activities in Becker's model are "black-boxed."

Lohmann (1993a, 1994) develops a model in which a large number of people may take costly political action (e.g., take part in a demonstration), either prior to a vote (the 1994 paper) or prior to a political leader's decision (the 1993a paper). By taking political action, the individual may be able to signal her private information to the other voters or to the political leader. Lohmann (1993b) carries out a welfare analysis of this model and shows that political action can be both under- and oversupplied in equilibrium, which also the present paper does. Related signaling models of lobbying are studied also in Lohmann (1995a,b). However, in contrast to the present paper, Lohmann's papers do not study the effects of a gradual increase in the number of political activists, nor do they relate to Tullock's model by, for example, calculating the degree of rent dissipation in the limit as the number of political activists goes to infinity.

There are two reasons why the model in the present paper can deal with an arbitrary number of rent seekers while still keeping the analysis tractable: (i) the information transmission is modeled as a disclosure game (i.e., rent seekers are allowed to withhold information but not to lie)[7] and (ii) the expected payoff of a typical rent seeker can be rewritten in a succinct form thanks to some algebraic manipulations (see Eq. (2) in Sect. 4).

The remainder of the paper is organized as follows. The next section provides a formal presentation of Tullock's model. In Sect. 3, the model with informational foundations is introduced. In Sect. 4, this model is solved and the first results are stated. Section 5 considers social welfare, and Sect. 6 illustrates the results using some numerical examples. Section 7 briefly concludes and discusses some possible extensions and variations of the model. Most of the proofs are relegated to some appendices.

2 The Tullock model

Tullock (1980) considers a model in which n identical economic agents, or rent seekers, try to win an indivisible prize, their valuation of which equals v (>0). Rent seeker i's probability of winning is given by

$$P_i(x_1, x_2, \ldots, x_n) = \begin{cases} \dfrac{x_i^R}{\sum_{j=1}^n x_j^R} & \text{if } \sum_{j=1}^n x_j^R \neq 0 \\ 1/n & \text{otherwise,} \end{cases}$$

where x_i (≥ 0) is this rent seeker's investment. The parameter R (>0) is a measure of how discriminatory the rent-seeking contest is: The larger is this parameter, the greater is the likelihood that a rent seeker who spends strictly more than his rivals will win. In particular, in the limit as R approaches infinity, a rent seeker who spends (perhaps just slightly) more than his rivals wins with

[7] Other contrubutions modeling lobbying as a disclosure game include Milgrom and Roberts (1986), Laffont and Tirole (1991, 1993, Chap. 11), Lagerlöf (1997), Dewatripont and Tirole (1999), Bennedsen and Feldmann (2002), and Lagerlöf and Heidhues (2005).

probability one. Similarly, in the limit as R approaches zero, each rent seeker wins with probability $1/n$, regardless of the relative magnitude of the x_i's (as long as they are all strictly positive). For the special case $R = 1$, a rent seeker's probability of winning equals his own investment divided by the sum of all rent seekers' investments. Tullock also assumed that the rent seekers simultaneously choose their x_i's with the objective of maximizing their expected net gain, $P_i(x_1, x_2, \ldots, x_n)\, v - x_i$, while taking the others' behavior as given.

It is a standard exercise to show that for $R \leq 1$ and for any finite n, there exists a unique pure strategy Nash equilibrium of Tullock's model. In this equilibrium, which is symmetric, each rent seeker's investment is given by

$$x^* = \frac{(n-1)\, Rv}{n^2}.$$

The *dissipation rate*, the amount of resources used in total by the rent seekers as a fraction of the value of the prize, therefore equals $nx^*/v = R\,(n-1)/n$. Hence, the dissipation rate is increasing in the number of rent seekers, and as the number of them goes to infinity the dissipation tends to R. As a consequence, under the assumption that the rent-seeking expenditures have no social value, more competition among rent seekers (i.e., a larger n) is always bad.

3 A model with informational foundations

There are n (≥ 2) ex ante identical firms and one regulator. The regulator can grant a monopoly on a market to one of the firms. Each firm is either "efficient" or "inefficient." If the firm that is granted the monopoly is efficient, then social welfare (we can think of this as total surplus or consumer surplus) is given by W_H; otherwise, if the firm is inefficient, social welfare is given by W_L, with $W_H > W_L$. The regulator cares about social welfare, and ideally he would like to grant the monopoly to a firm that is efficient. Initially, however, the regulator faces uncertainty about whether a particular firm is efficient or inefficient. The prior probability that a firm is efficient is given by μ, where $\mu \in (0, 1)$. Each firm knows itself whether it is efficient or inefficient,[8] but it faces the same uncertainty about the other firms as does the regulator. The realizations of these random events are independent.

An efficient firm that is granted the monopoly earns the monopoly profits π_H. An inefficient firm that is granted the monopoly earns the monopoly profits π_L, with $\pi_H > \pi_L > 0$. A firm that is not granted the monopoly earns some "outside profits," which are normalized to zero. Hence, any firm, regardless of whether it is efficient or inefficient, wants to be granted the monopoly.

Although each firm at the outset knows whether it is efficient or inefficient, the information it has is assumed to be soft (i.e., non-verifiable). This means

[8] An alternative assumption would be that a firm faces the same uncertainty about its own efficiency as do the other firms and the regulator. That assumption would yield very similar results, however.

that an efficient firm will not be able to simply tell the regulator about the fact that it is efficient – such a claim would not be credible since an inefficient firm has an incentive to make the same claim. The firms can, however, in addition to their soft information, gather information that is hard (i.e., verifiable). Doing this is costly, but if a firm finds hard information showing that it is efficient, it can show this to the regulator who will then be more likely to grant the monopoly to that particular firm.

Formally, conditional on knowing whether it is efficient or inefficient, each firm i first chooses how much to spend on information gathering,[9] $y_{ij} \in \Re_+$ (where the subindex $j = H, L$ indicates whether the firm is efficient or inefficient, respectively). This level of expenditures determines a probability $\varphi\left(y_{ij}\right) \in \left[0, 1\right]$, where $\varphi\left(\cdot\right)$ is a twice differentiable, increasing, and strictly concave function satisfying $\varphi\left(0\right) = 0$ and $\lim_{y_{ij} \to 0} \varphi'\left(y_{ij}\right) = \infty$. With this probability, the firm finds hard information as to whether it is efficient or inefficient; with the complementary probability $1 - \varphi\left(y_{ij}\right)$, the firm does not find anything. Since the information is hard, the firm can, if it so likes, show the evidence to the regulator, in which case the regulator learns (for sure) whether the firm is efficient or inefficient. The firms choose y_{ij} simultaneously, and neither this choice nor the fact whether they find hard information or not can be observed by the other firms or by the regulator.

The regulator is a Bayesian updater. Given that some of the firms have shown him hard information that they are efficient and the others have not, the regulator grants the monopoly to a firm whose likelihood of being efficient, as perceived by the regulator, is at least as high as the likelihood for any other firm. In the equilibrium of the model, this must be a firm that has shown the regulator hard information, as long as there is such a firm.[10] It is assumed that the regulator chooses which firm of those who have submitted hard information to grant the monopoly by using a fair lottery; hence, if k firms have submitted hard information, each of them is granted the monopoly with probability $1/k$. Similarly, in case no firm has submitted hard information, the monopoly is allocated using a fair lottery among all n firms, so that each is granted the monopoly with probability $1/n$.

4 Analysis

Given that the firms in the model are ex ante identical, it is natural to consider equilibria in which all firms' behavior are the same. I will thus confine attention to symmetric equilibria of the model.

[9] Given that firms have soft information from the outset and that they gather only hard information, one could have used the term "evidence production" rather than "information gathering." I will stick to the latter term, though.

[10] For these firms are efficient with probability one, and the others are efficient with a strictly lower probability. To see that this must be the case, remember that $\mu < 1$. Moreover, the fact that a firm has not presented hard information speaks against the possibility that it is efficient, since in equilibrium the inefficient firms have a lower (in fact zero) incentive to gather information.

As noted above, a firm that knows that it is inefficient will not expend resources trying to find hard evidence about this; thus, $y_{iL} = 0$ for all i. In order to characterize the equilibrium choice of y_{iH}, we need to find an expression for firm i's expected payoff given that it itself chooses the investment level y_{iH} and that it believes its efficient rivals all choose, say, the investment level $\widehat{y}$. If firm i as well as exactly $k \in \{0, 1, 2, \ldots, n-1\}$ of its rivals are both efficient and successful in their investigations, then firm i is granted the monopoly with probability $1/(1+k)$. Moreover, the probability that exactly k of the $n-1$ rivals are efficient and successful, given that they all chose $\widehat{y}$, is given by

$$\binom{n-1}{k} \left[\mu\varphi\left(\widehat{y}\right)\right]^k \left[1-\mu\varphi\left(\widehat{y}\right)\right]^{n-1-k}.$$

This means that firm i's expected profit at the stage where it is about to choose y_{iH} can be written as

$$V_{iH}\left(y_{iH}, \widehat{y}\right) = \varphi\left(y_{iH}\right) \sum_{k=0}^{n-1} \binom{n-1}{k} \left[\mu\varphi\left(\widehat{y}\right)\right]^k \left[1-\mu\varphi\left(\widehat{y}\right)\right]^{n-1-k} \frac{\pi_H}{k+1}$$
$$+ \left[1-\varphi\left(y_{iH}\right)\right] \left[1-\mu\varphi\left(\widehat{y}\right)\right]^{n-1} \frac{\pi_H}{n} - y_{iH}. \tag{1}$$

For large n, this expression consists of a very long series of terms. We can write it much more succinctly, however, thanks to the following relationship (see Appendix A for the derivation):

$$\sum_{k=0}^{n-1} \binom{n-1}{k} \frac{\left[\mu\varphi\left(\widehat{y}\right)\right]^k \left[1-\mu\varphi\left(\widehat{y}\right)\right]^{n-1-k}}{k+1} = \frac{1-\left[1-\mu\varphi\left(\widehat{y}\right)\right]^n}{\mu\varphi\left(\widehat{y}\right) n}. \tag{2}$$

The expression in (2) is the expected likelihood that firm i will be granted the monopoly, at the stage when it knows that it itself is efficient and has found hard information about this. Using (2) in (1) and then rearranging, we have

$$V_{iH}\left(y_{iH}, \widehat{y}\right) = P\left(y_{iH}, \widehat{y}\right) \pi_H - y_{iH}, \tag{3}$$

where

$$P\left(y_{iH}, \widehat{y}\right) \equiv \frac{\left[1-\left[1-\mu\varphi\left(\widehat{y}\right)\right]^{n-1}\right] \varphi\left(y_{iH}\right)}{\mu\varphi\left(\widehat{y}\right) n} + \frac{\left[1-\mu\varphi\left(\widehat{y}\right)\right]^{n-1}}{n}. \tag{4}$$

The function $P(\cdot)$ can be thought of as a probability-of-winning function, analogous to the one in Tullock's model. In contrast to Tullock's model, however, here we have $P(0, \widehat{y}) > 0$: a firm can win the monopoly even if it does not expend any resources at all. The reason is that if all other firms either are inefficient or are efficient but fail to find hard information, a firm wins the monopoly with probability $1/n$.[11]

There are also other, strategically more important, differences between this model and Tullock's. In the present model, the cross-derivative of the objective function $V_{iH}(y_{iH}, \widehat{y})$ is zero for $n = 2$, which means that each firm has a strictly dominant strategy; and it is negative for $n \geq 3$ (for the proof of this, see Appendix B), which means that the firms' choice variables are (loosely speaking) strategic substitutes.[12] In contrast, in Tullock's model the rent seekers' choice variables are (globally) neither strategic complements nor strategic substitutes, as the sign of the cross derivative depends on the relative magnitude of the own and the rivals' choice.

Moreover, the term in (3) that corresponds to v, the value of winning the prize in Tullock's model, is not (as one might have expected without giving it much thought) equal to $E\pi = (1-\mu)\pi_L + \mu\pi_H$, but to π_H. This is *not* just because the objective function in (3) is calculated conditional on knowing that the firm is efficient; we would get the same result if we instead had assumed that the firms do not know whether they are efficient or inefficient when choosing their rent-seeking investment (as suggested in footnote 8). The reason why π_H, and not $E\pi$, is what matters is that the investment choice is made with an eye towards the firm's profits *conditional on being efficient* – the point with expending y_{iH} is that it may help the firm prove that it is efficient. (The same logic explains why $y_{iL} = 0$.) Similarly, in an alternative model in which the firms did not know initially whether they were efficient or inefficient, the choice of investment level would be a function of π_H, not of $E\pi$.

Firm i's problem is to maximize (3) with respect to y_{iH}, taking the other firms' behavior (i.e., $\widehat{y}$) as given. The first-order condition to this problem is

$$\frac{\partial V_{iH}(y_{iH}, \widehat{y})}{\partial y_{iH}} = \frac{\left[1 - \left[1 - \mu\varphi(\widehat{y})\right]^{n-1}\right]\varphi'(y_{iH})}{\mu\varphi(\widehat{y})\, n}\pi_H - 1 = 0. \tag{5}$$

The assumption that $\lim_{y_{iH}\to 0}\varphi'(y_{iH}) = \infty$ guarantees that $y_{iH} = 0$ is not optimal. Also, let us assume that the parameters are such that the y_{iH} that solves (5) satisfies $\varphi(y_{iH}) < 1$ (for example, that π_H is not too large).

In a symmetric equilibrium we must have $y_{iH} = \widehat{y} = y^*$. Plugging this into (5), we obtain the following result (the uniqueness claim is proven in Appendix C).

Proposition 1 *For any $n \geq 2$, there exists a pure strategy symmetric equilibrium. Within the class of symmetric equilibria, this equilibrium is unique, and it is*

[11] From (4) we see that $P(0, \widehat{y})$ is equal to the product of $1/n$ and $\left[1 - \mu\varphi(\widehat{y})\right]^{n-1}$, the latter being the probability that all other $n-1$ firms either are inefficient or are efficient but fail to find hard information.

[12] This is only "loosely speaking" because we have assumed that the value of all other firms' choice variables, $\widehat{y}$, are identical (before and after the infinitesimal change).

characterized by

$$\frac{1-\left[1-\mu\varphi\left(y^{*}\right)\right]^{n-1}}{\mu\varphi\left(y^{*}\right)n}\pi_{H}=\frac{1}{\varphi'\left(y^{*}\right)}. \tag{6}$$

We also have the following comparative statics results (proven in Appendix D).

Proposition 2 *The equilibrium investment level y^* is strictly increasing in π_H, and (for $n \geq 3$) it is strictly decreasing in μ.*

The second result is the only one that is not obvious. The reason why the equilibrium investment level is decreasing in μ is that (for $n \geq 3$) the players' choice variables are strategic substitutes, and a larger μ means that it is more likely that firm *i*'s typical rival is of a type who chooses a positive investment level.

Among other things, we are interested in what happens to aggregate expenditures as n goes to infinity. To investigate this, we first need the following results (proven in Appendix E).

Lemma 1 *As $n \to \infty$, $y^* \to 0$.*

Lemma 2 *As $n \to \infty$, $\left[1-\mu\varphi\left(y^{*}\right)\right]^{n-1} \to 0$.*

Lemma 2 tells us that the probability that (all other) $n-1$ firms lack hard evidence showing that that they are efficient goes to zero as n goes to infinity.

Define expected total expenditures as $\mathrm{TE}\left(y^*\right) \equiv n\mu y^*$. By (6),

$$\left[1-\left[1-\mu\varphi\left(y^{*}\right)\right]^{n-1}\right]\pi_{H}\eta\left(y^{*}\right)=n\mu y^{*}=\mathrm{TE}\left(y^{*}\right), \tag{7}$$

where

$$\eta\left(y\right)\equiv\frac{\varphi'\left(y\right)y}{\varphi\left(y\right)}$$

is the elasticity associated with the function $\varphi\left(\cdot\right)$.[13] (Note that by concavity of $\varphi\left(\cdot\right)$, $\eta\left(y\right) < 1$ for all y.) By taking the limit of (7), using Lemmas 1 and 2 and the notation $\eta\left(0\right) \equiv \lim_{y\to 0}\eta\left(y\right)$, we obtain the following.

Proposition 3 *In the limit as the number of firms goes to infinity, expected total expenditures equal the monopoly profits for an efficient firm times the elasticity $\eta\left(y\right)$ evaluated at zero*:

$$\lim_{n\to\infty}\mathrm{TE}\left(y^{*}\right)=\eta\left(0\right)\pi_{H}.$$

[13] Note that the inverse of $\varphi\left(\cdot\right)$, call it $C\left(\cdot\right)$, gives us the cost a rent seeker must incur in order to benefit from a particular probability of finding hard information. The elasticity $\eta\left(y\right)$ can therefore be interpreted as the inverse of the elasticity associated with the function $C\left(\cdot\right)$, or the inverse of the cost elasticity.

Apparently, the parameter R in Tullock's model corresponds to $\eta(0)$ in the present model, in the sense that this parameter tells us what fraction of the rent seekers' prize will be dissipated by expenditures as the number of rent seekers goes to infinity.

5 Welfare

Suppose all n firms invest the same amount, y, if they are efficient and zero otherwise. Given y, let $\mathrm{EW}(y)$ be defined as the ex ante expected social welfare, net of expected total expenditures:

$$\begin{aligned}\mathrm{EW}(y) = \left[1-(1-\mu\varphi(y))^n\right] W_H + (1-\mu\varphi(y))^n \left[\widehat{\mu} W_H + (1-\widehat{\mu}) W_L\right] \\ -\mathrm{TE}(y), \end{aligned} \tag{8}$$

where

$$\widehat{\mu} \equiv \frac{\mu\left[1-\varphi(y)\right]}{1-\mu\varphi(y)} \tag{9}$$

is the probability that a firm that has not provided hard information showing that it is efficient nevertheless is efficient (this can be derived by using Bayes' rule). Note that, quite intuitively, $\widehat{\mu} < \mu$ for any $\varphi(y) > 0$.

The first term in (8) is given by the product of (i) W_H (social welfare if having an efficient firm) and (ii) the probability that at least one of the n firms indeed is efficient and finds hard information showing this. The second term in (8) is given by the product of (i) the probability that none of the n firms is efficient and finds hard information showing this and (ii) expected social welfare given that the firm being granted the monopoly is efficient with probability $\widehat{\mu}$. The third term in (8) is the expected total rent-seeking expenditures. Substituting (9) into (8) and simplifying we have

$$\mathrm{EW}(y) = W_H - (1-\mu)(1-\mu\varphi(y))^{n-1}\Delta W - \mathrm{TE}(y), \tag{10}$$

where $\Delta W \equiv W_H - W_L$.

We may ask two questions about $\mathrm{EW}(y)$. First, if a planner could choose the y that maximizes $\mathrm{EW}(y)$, call it y^w, how would this relate to y^*? That is, in the equilibrium of the model, does an individual firm spend too much or too little resources on rent seeking, relative to the socially desirable level? Second, if we evaluate $\mathrm{EW}(y)$ at $y = y^*$, will this expression be increasing or decreasing in the number of firms, n? That is, is more competition among the rent seekers good or bad for welfare in this model where the influence activities have informational foundations?

To answer the first question, let us differentiate $\mathrm{EW}(y)$ and set the resulting expression equal to zero, thus obtaining the following first-order condition, which implicitly defines y^w:

$$(1-\mu)(n-1)\left(1-\mu\varphi\left(y^w\right)\right)^{n-2}\varphi'\left(y^w\right)\Delta W - n = 0. \tag{11}$$

The assumption that $\lim_{y\to 0}\varphi'(y)=\infty$ guarantees that $y^w=0$ is not optimal. Let us also assume that the parameters are such that the y^w that solves (11) satisfies $\varphi(y^w)<1$ (for example, that ΔW is not too large).

In general, y^w may be either smaller or larger than y^*. However, comparing (11) and the first-order condition that defines y^*, (6), we can make the following three observations: (i) The "prize" that matters for society is ΔW: the difference in social welfare between having an efficient firm and having an inefficient firm. In contrast, as we noted above, a firm that chooses y_{iH} cares about π_H, its profits conditional on being efficient.[14] As a consequence, even if society's and an individual firm's incentives were aligned in the sense that $W_H=\pi_H$ and $W_L=\pi_L$, the firm would still invest too much if π_L is sufficiently close to π_H , regardless of the values of the other parameters. (ii) For $n=2$, the two first-order conditions can be written as

$$\frac{(1-\mu)\,\Delta W}{2}=\frac{1}{\varphi'(y^w)} \tag{12}$$

and

$$\frac{\pi_H}{2}=\frac{1}{\varphi'(y^*)}. \tag{13}$$

This means that, for this case,

$$y^w \gtreqless y^* \text{ as } (1-\mu)\,\Delta W \gtreqless \pi_H. \tag{14}$$

Consistent with observation (i), we see that π_L does not enter the condition in (14), only ΔW and π_H do. In addition (12) and (13) tell us that whereas the firm's choice does not depend on μ, society's optimal choice of rent seeking investment is decreasing in μ. In particular, for any values of ΔW and π_H (e.g., for ΔW being much larger than π_H) the firm will spend too much resources on rent seeking if the probability μ is sufficiently close to one. Intuitively, if μ is close to unity, society knows that a firm that it picks just by random is likely to be efficient; thus, it is in society's interest that only a very small amount of resources is spent on rent seeking. In contrast, the firm does not take this into account; it expends the same amount on rent seeking for all values of μ . (iii) Again making use of the first-order conditions that define y^* and y^w, we see that (now for any arbitrary n) in the limit as μ approaches 1, y^w approaches 0 whereas y^* approaches some strictly positive number (the intuition being the same as the one discussed under observation (ii)).

In order to answer our second question – whether $\mathrm{EW}(y^*)$ is increasing or decreasing in n – we can evaluate (10) at $y=y^*$ and then rewrite it using (7),

[14] As discussed previously, the reason why the firm cares about π_H rather than $E\pi$ is not just that it knows that it is efficient; it would make its choice contingent on being efficient also if it made the choice at an ex ante stage. The reason for this is that the point with expending y_{iH} is that it may help the firm prove that it is efficient.

thus obtaining

$$\mathrm{EW}\left(y^*\right) = \mu W_H + (1-\mu)\, W_L + \mathrm{TE}\left(y^*\right)\left[\frac{(1-\mu)\,\Delta W}{\pi_H \eta\,(y^*)} - 1\right].$$

It follows that, if the elasticity $\eta\,(y)$ is constant (i.e., $\eta\,(y) = \eta$), then

$$\frac{\partial \mathrm{EW}\,(y^*)}{\partial n} = \left[\frac{(1-\mu)\,\Delta W}{\pi_H \eta} - 1\right]\frac{\partial \mathrm{TE}\,(y^*)}{\partial n}. \tag{15}$$

We therefore have the following result.

Proposition 4 *If the elasticity $\eta\,(y)$ is constant and if* $\mathrm{TE}\,(y^*)$ *is increasing in n, then*

$$\frac{\partial EW\,(y^*)}{\partial n} \gtreqless 0 \text{ as } (1-\mu)\,\Delta W \gtreqless \eta \pi_H.$$

The important implication of Proposition 4 is that, even if $\mathrm{TE}\,(y^*)$ is increasing in n,[15] $\mathrm{EW}\,(y^*)$ is not necessarily decreasing in n. This result stands in sharp contrast to Tullock's model (although there, the fact that a larger number of rent seekers is bad for welfare follows immediately from the assumption that more expenditures is a bad thing). In particular, provided $\mathrm{TE}\,(y^*)$ is increasing in n, $\mathrm{EW}\,(y^*)$ is increasing in n as well if $(1-\mu)\,\Delta W > \eta\pi_H$. This inequality will be satisfied if: (i) ΔW is large, so that society cares a lot about whether an efficient or an inefficient firm is granted the monopoly; (ii) μ is small, so that the prior probability that a firm is efficient firm is low (which makes it more desirable to have many firms to choose among); (iii) π_H is small, so that an efficient firm does not have a lot to gain from being granted the monopoly; and (iv) η is small, so that the amount of expected total rent-seeking expenditures with many firms is small.

When the inequality is reversed, so that $(1-\mu)\,\Delta W < \eta\pi_H$, $\mathrm{EW}\,(y^*)$ is decreasing in n whenever $\mathrm{TE}\,(y^*)$ is increasing in n, just as is assumed in Tullock's model. For the special case $(1-\mu)\,\Delta W = \eta\pi_H$, $\mathrm{EW}\,(y^*)$ is constant with respect to n; in particular, $\mathrm{EW}\,(y^*)$ takes the same value evaluated at $n = 1$ as it does in the limit as n goes to infinity:[16]

$$\mathrm{EW}\left(y^*\right)\big|_{n=1} = \mu W_H + (1-\mu)\, W_L = W_H - \eta\pi_H = \lim_{n\to\infty} \mathrm{EW}\left(y^*\right).$$

[15] Although it remains to be proven analytically, $\mathrm{TE}\left(y^*\right)$ appears to be increasing in n for all parameter values of the model, at least as long as the elasticity $\eta\,(y)$ is constant (see the examples in the next section).

[16] The model does not formally allow for $n = 1$. However, it is straightforward to see that a firm that does not have any rivals will not spend any resources on rent seeking: $y^*\,|_{n=1} = 0$.

6 Numerical examples

In this section I will study some numerical examples that illustrate the results in the two previous sections. Rewriting the first-order condition that defines y^*, (6), we have

$$y^* = \left[1 - \left[1 - \mu\varphi\left(y^*\right)\right]^{n-1}\right] \frac{\eta\left(y^*\right)\pi_H}{\mu n}. \tag{16}$$

Similarly, rewriting (11) yields

$$y^w = \left(1 - \mu\varphi\left(y^w\right)\right)^{n-2} \frac{(1-\mu)(n-1)\,\eta\left(y^w\right)\varphi\left(y^w\right)\Delta W}{n}. \tag{17}$$

Using (16) and (17) – and given functional forms for $\varphi(\cdot)$ and values of the parameters – we can easily (using, for example, a spreadsheet program like Excel) calculate values of y^* and y^w by first guessing a value, substitute this into the right-hand side of (16) and (17), respectively, thereby obtaining a new guess, which again can be substituted into the right-hand side of the relevant equation. After sufficiently many iterations, this procedure will give us a good approximation of the unique solution to the equation.

As an illustration, let us assume that

$$\varphi(y) = \frac{\sqrt{y}}{16},$$

which implies that $\eta(y)$ is constant and equal to 0.5. Moreover, let $\mu = 0.5$, $W_H = 500$, and $W_L = 100$ (so that $\Delta W = 400$). Given these functional forms and parameter values, y^* and y^w have, for various values of n, been calculated using six alternative values of π_H:[17]

$$\pi_H \in \{100, 200, 400, 600, 800, 1000\}.$$

In addition, using the calculated values of y^* and y^w, the associated levels of expected total rent-seeking expenditures and of expected welfare have been calculated. The six different values of π_H were chosen to make sure that: for $\pi_H \in \{100, 200\}$, $\mathrm{EW}(y^*)$ is increasing in n whenever $\mathrm{TE}(y^*)$ is increasing in n; for $\pi_H \in \{600, 800, 1000\}$, $\mathrm{EW}(y^*)$ is decreasing in n whenever $\mathrm{TE}(y^*)$ is increasing in n; and for $\pi_H = 400$, $\mathrm{EW}(y^*)$ is constant with respect to n.

The results are reported in Table 1, and graphs of $\mathrm{EW}(y^*)$ as a function of n for the six different values of π_H are shown in Fig. 1. The table shows that for

[17] It has also been verified that, given the chosen parameters, the assumptions that $\varphi\left(y^*\right) < 1$ and $\varphi\left(y^w\right) < 1$ are satisfied.

Table 1 Numerical examples

n		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
y*	$\pi_H = 100$	0.00	2.44	4.07	4.78	5.06	5.11	5.06	4.95	4.82	4.67	4.52	4.37	4.22	4.08	3.95	3.82	3.70	3.59	3.48	3.37
TE(y*)		0.0	2.4	6.1	9.6	12.6	15.3	17.7	19.8	21.7	23.4	24.9	26.2	27.5	28.6	29.6	30.6	31.5	32.3	33.0	33.7
EW(y*)		300.0	307.3	318.3	328.7	337.9	346.0	353.1	359.4	365.0	370.1	374.6	378.7	382.4	385.8	388.9	391.7	394.4	396.8	399.1	401.2
y*	$\pi_H = 200$	0.00	9.77	15.30	16.90	16.94	16.38	15.62	14.80	14.01	13.26	12.56	11.92	11.33	10.79	10.30	9.84	9.42	9.03	8.68	8.34
TE(y*)		0.0	9.8	23.0	33.8	42.3	49.1	54.7	59.2	63.0	66.3	69.1	71.5	73.7	75.5	77.2	78.7	80.1	81.3	82.4	83.4
EW(y*)		300.0	309.8	323.0	334.3	343.5	351.3	357.9	363.6	368.6	373.1	377.1	380.8	384.1	387.1	390.0	392.6	395.0	397.3	399.4	401.4
y*	$\pi_H = 400$	0.00	39.06	54.37	54.45	50.80	46.57	42.60	39.08	36.00	33.32	30.98	28.93	27.12	25.51	24.07	22.78	21.62	20.57	19.61	18.74
TE(y*)		0.0	39.1	81.5	108.9	127.0	139.7	149.1	156.3	162.0	166.6	170.4	173.6	176.3	178.6	180.5	182.3	183.8	185.1	186.3	187.4
EW(y*)		300.0	300.0	300.0	300.0	300.0	300.0	300.0	300.0	300.0	300.0	300.0	300.0	300.0	300.0	300.0	300.0	300.0	300.0	300.0	300.0
y*	$\pi_H = 600$	0.00	87.89	109.36	101.87	90.83	80.76	72.27	65.21	59.30	54.31	50.06	46.41	43.23	40.45	38.00	35.82	33.87	32.12	30.54	29.10
TE(y*)		0.0	87.9	164.0	203.7	227.1	242.3	253.0	260.8	266.8	271.6	275.3	278.4	281.0	283.2	285.0	286.6	287.9	289.1	290.1	291.0
EW(y*)		300.0	270.7	245.3	232.1	224.3	219.2	215.7	213.1	211.1	209.5	208.2	207.2	206.3	205.6	205.0	204.5	204.0	203.6	203.3	203.0
y*	$\pi_H = 800$	0.00	156.25	174.85	154.16	133.29	116.24	102.64	91.70	82.77	75.36	69.14	63.84	59.27	55.31	51.83	48.75	46.02	43.57	41.36	39.37
TE(y*)		0.0	156.2	262.3	308.3	333.2	348.7	359.3	366.8	372.5	376.8	380.3	383.0	385.3	387.2	388.7	390.0	391.2	392.1	393.0	393.7
EW(y*)		300.0	221.9	168.9	145.8	133.4	125.6	120.4	116.6	113.8	111.6	109.9	108.5	107.4	106.4	105.6	105.0	104.4	103.9	103.5	103.2
y*	$\pi_H = 1000$	0.00	244.14	247.03	208.75	176.63	152.04	133.09	118.16	106.15	96.30	88.08	81.14	75.19	70.04	65.54	61.58	58.07	54.92	52.10	49.55
TE(y*)		0.0	244.1	370.5	417.5	441.6	456.1	465.8	472.6	477.7	481.5	484.5	486.8	488.7	490.3	491.6	492.7	493.6	494.3	495.0	495.5
EW(y*)		300.0	153.5	77.7	49.5	35.1	26.3	20.5	16.4	13.4	11.1	9.3	7.9	6.8	5.8	5.1	4.4	3.9	3.4	3.0	2.7
y^w		0.00	9.77	13.59	13.49	12.42	11.21	10.09	9.11	8.26	7.53	6.90	6.36	5.88	5.46	5.09	4.76	4.46	4.20	3.96	3.74
TE(y^w)		0.0	9.8	20.4	27.0	31.0	33.6	35.3	36.4	37.2	37.7	38.0	38.1	38.2	38.2	38.2	38.1	37.9	37.8	37.6	37.4
EW(y^w)		300.0	309.8	323.0	334.3	343.5	351.3	357.9	363.6	368.6	373.1	377.1	380.8	384.1	387.1	390.0	392.6	395.0	397.3	399.4	401.4

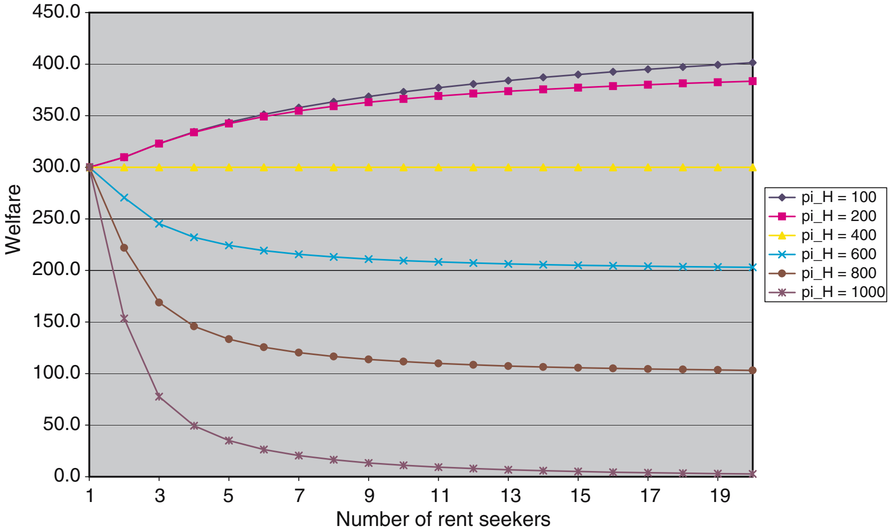

Fig. 1 Welfare with eq. rent-seeking levels

all six cases $\mathrm{TE}(y^*)$ is increasing in n. We also see, however, that y^* is, in all our examples, not monotone in n.

In summary, Table 1 and Fig. 1 illustrate the insight from Proposition 4 that in this model of rent seeking with informational foundations, expected welfare is not necessarily increasing in the number of rent seekers just because the total rent-seeking expenditures are. Competition among rent seekers may be good. In particular, we obtained this result for relatively low values of π_H, as in those cases the rent seekers' benefit from rent seeking is small relative to society's.

7 Summary and concluding remarks

This paper has developed a model of rent seeking with informational foundations and with an arbitrary number of rent seeking firms. In general in this model, the firms' incentives to spend resources on rent seeking will obviously differ from society's, as the firms care about their own expected profits and not about social welfare. What the analysis revealed and which is less obvious, however, is that the firms will choose their rent seeking investments with an eye toward their profits *conditional on being an efficient* (as opposed to inefficient) firm, the reason being that the point with doing rent seeking is that this may provide evidence that the firm is efficient. Society, on the other hand, would like the choice of rent seeking expenditures to be made with an eye toward the *difference* in social welfare between having an efficient firm and having an inefficient firm – for it is this difference that society can gain by, thanks to the additional information, being able to grant the monopoly to an efficient firm rather than an inefficient one. To the extent that this difference is smaller than the firm's profits conditional on being efficient (which seems to be a natural possibility, as the latter is a profit *level* and not a difference), the rent seekers will tend to – in the spirit of the rent-seeking literature – have a too strong incentive to invest in rent seeking from a social welfare point of view.

The analysis also showed that in the limit as the number of firms approach infinity, the total expected amount of rent-seeking expenditures equal a constant $\eta(0)$ times the perceived prize, where this constant is equal to the inverse of the elasticity of the cost function associated with information acquisition (evaluated at zero). Thus, the constant $\eta(0)$ is a direct analogue to the parameter R in Tullock's model. Most importantly, the microfoundations of the model in the present paper allowed us to conduct a meaningful welfare analysis. In particular, I derived a condition that determines whether expected welfare and total rent-seeking expenditures move in opposite directions or in the same direction when the number of rent seekers increases. For many configurations of the parameters, expected welfare increases as the number of rent seekers becomes larger, which means that competition among rent seekers may be welfare-enhancing.

Let me conclude by briefly discussing a couple of possible extensions and variations of the model. First, as already mentioned in footnote 8, instead of assuming that each firm at the outset has access to full (but only soft) information about its own efficiency, one could say that a firm faces the same uncertainty about its own efficiency as do the other firms and the regulator. One can show, however, that – because of the simple, linear structure of the model – this assumption would in many regards yield very similar results.[18] Second, I assumed that the firms can, at a cost, obtain hard evidence that confirms their true type. Alternatively, one could have assumed that the inefficient firms can in addition fabricate (false) evidence showing that they are efficient, and that doing this is costly for the firms. In such a model, expenditures on fabrication of evidence cannot have any social value. Therefore, we should expect that the possibility of fabrication would add to the social costs of rent seeking. Because of the same reason we should also expect the set of circumstances under which expected welfare increases as total rent-seeking expenditures increase to be smaller.[19]

[18] The fact that the analysis under this alternative assumption would be very similar is quite significant. The present model could be criticized for allowing the regulator to use only a very limited set of instruments: if the regulator were able to commit to a somewhat richer mechanism, he would easily be able to implement the first best. However, under the alternative assumption that each firm faces the same uncertainty about its own efficiency as do the other firms and the regulator, implementing the first best would be much less straightforward.

[19] Lagerlöf and Heidhues (2005, Section 6.4 and Appendix A) discuss the consequences of this kind of assumption in the context of a model that is similar to the one in the present paper (but with a single rent seeker).

Appendix A: Proof of Eq. (2)

Since $\binom{n-1}{k} = \binom{n}{k+1}\frac{k+1}{n}$, we can write

$$\begin{aligned}
&\sum_{k=0}^{n-1}\binom{n-1}{k}\frac{(\mu\varphi(\widehat{y}))^k(1-\mu\varphi(\widehat{y}))^{n-1-k}}{k+1} \\
&\quad = \frac{1}{n}\sum_{k=0}^{n-1}\binom{n}{k+1}(\mu\varphi(\widehat{y}))^k(1-\mu\varphi(\widehat{y}))^{n-1-k} \\
&\quad = \frac{1}{\mu\varphi(\widehat{y})\,n}\sum_{k=0}^{n-1}\binom{n}{k+1}(\mu\varphi(\widehat{y}))^{k+1}(1-\mu\varphi(\widehat{y}))^{n-(k+1)}.
\end{aligned}$$

Defining $m \equiv k+1$, we can rewrite the last line as

$$\begin{aligned}
&\frac{1}{\mu\varphi(\widehat{y})\,n}\sum_{m=1}^{n}\binom{n}{m}(\mu\varphi(\widehat{y}))^m(1-\mu\varphi(\widehat{y}))^{n-m} \\
&= \frac{1}{\mu\varphi(\widehat{y})\,n}\sum_{m=0}^{n}\binom{n}{m}(\mu\varphi(\widehat{y}))^m(1-\mu\varphi(\widehat{y}))^{n-m} - \frac{1}{\mu\varphi(\widehat{y})\,n}\binom{n}{0}(1-\mu\varphi(\widehat{y}))^n \\
&= \frac{1-(1-\mu\varphi(\widehat{y}))^n}{\mu\varphi(\widehat{y})\,n},
\end{aligned}$$

where the last equality makes use of $\sum_{m=0}^{n}\binom{n}{m}(\mu\varphi(\widehat{y}))^m(1-\mu\varphi(\widehat{y}))^{n-m} = 1$ and $\binom{n}{0} = 1$. □

Appendix B: Proof of the claim about strategic substitutes for $n \geq 3$

We need to show that $\partial^2 V_{iH}(y_{iH},\widehat{y})/\partial y_{iH}\partial\widehat{y} < 0$ for $n \geq 3$. Differentiating (3) yields

$$\begin{aligned}
&\frac{\partial^2 V_{iH}(y_{iH},\widehat{y})}{\partial y_{iH}\partial\widehat{y}} \\
&\quad = \frac{\mu\varphi'(\widehat{y})(n-1)\left[1-\mu\varphi(\widehat{y})\right]^{n-2}\varphi(\widehat{y}) - \varphi'(\widehat{y})\left[1-\left[1-\mu\varphi(\widehat{y})\right]^{n-1}\right]}{\left[\varphi(\widehat{y})\right]^2} \\
&\qquad \times\frac{\varphi'(y_{iH})}{\mu n} \\
&\quad = \varphi'(\widehat{y})\left[\left[1-\mu\varphi(\widehat{y})\right]^{n-2}\left[(n-2)\mu\varphi(\widehat{y})+1\right]-1\right]\times\frac{\varphi'(y_{iH})}{\mu n\left[\varphi(\widehat{y})\right]^2}.
\end{aligned}$$

We thus need to show that

$$\left[1-\mu\varphi\left(\widehat{y}\right)\right]^{n-2}\left[\left(n-2\right)\mu\varphi\left(\widehat{y}\right)+1\right]<1$$

for all $n \geq 3$. This will be done by mathematical induction. One can easily show that the inequality holds for $n = 3$. Suppose it holds for some $n = m$ (≥ 3). The left-hand side of the inequality evaluated at $n = m + 1$ can be written as

$$\begin{aligned}
&\left[1-\mu\varphi\left(\widehat{y}\right)\right]^{m-1}\left[\left(m-1\right)\mu\varphi\left(\widehat{y}\right)+1\right]\\
&\quad=\left[1-\mu\varphi\left(\widehat{y}\right)\right]\left[1-\mu\varphi\left(\widehat{y}\right)\right]^{m-2}\left[\left(m-2\right)\mu\varphi\left(\widehat{y}\right)+1+\mu\varphi\left(\widehat{y}\right)\right]\\
&\quad=\left[1-\mu\varphi\left(\widehat{y}\right)\right]\left[1-\mu\varphi\left(\widehat{y}\right)\right]^{m-2}\left[\left(m-2\right)\mu\varphi\left(\widehat{y}\right)+1\right]+\left[1-\mu\varphi\left(\widehat{y}\right)\right]^{m-1}\mu\varphi\left(\widehat{y}\right)\\
&\qquad<\left[1-\mu\varphi\left(\widehat{y}\right)\right]+\left[1-\mu\varphi\left(\widehat{y}\right)\right]^{m-1}\mu\varphi\left(\widehat{y}\right)\\
&\qquad<\left[1-\mu\varphi\left(\widehat{y}\right)\right]+\left[1-\mu\varphi\left(\widehat{y}\right)\right]\mu\varphi\left(\widehat{y}\right)=1-\left[\mu\varphi\left(\widehat{y}\right)\right]^{2}<1,
\end{aligned}$$

where the first inequality follows from the induction assumption and the second and third by the fact that $\mu\varphi\left(\widehat{y}\right) < 1$. □

Appendix C: Proof of Proposition 1

The existence and characterization of the symmetric equilibrium is proven in the text. To see that the equilibrium is unique, first notice that the right-hand side of (6) is strictly increasing in y^*, due to the concavity of $\varphi(\cdot)$. Second, differentiating the left-hand side of (6) with respect to y^* yields $-\left[1-g\left(\mu,\varphi\left(y^*\right),n\right)\right]\pi_H\varphi'\left(y^*\right) / \left[\varphi\left(y^*\right)\right]^2\mu n$, where

$$g\left(\mu,\varphi\left(y^*\right),n\right)=\left(1-\mu\varphi\left(y^*\right)\right)^{n-2}\left[1+\left(n-2\right)\mu\varphi\left(y^*\right)\right].$$

It thus suffices to show that $g\left(\mu,\varphi\left(y^*\right),n\right)\leq 1$ for all $y^*\in\left[0,1\right]$. Differentiating g with respect to y^* yields

$$\begin{aligned}
\frac{\partial g\left(\mu,\varphi\left(y^*\right),n\right)}{\partial y^*}&=-\mu^2\varphi'\left(y^*\right)\varphi\left(y^*\right)\left(n-1\right)\left(n-2\right)\left(1-\mu\varphi\left(y^*\right)\right)^{n-3}\\
&\leq 0 \text{ for all } \varphi\left(y^*\right)\in\left[0,1\right].
\end{aligned}$$

Hence, for all $\varphi\left(y^*\right)\in\left[0,1\right]$, $g\left(\mu,\varphi\left(y^*\right),n\right)\leq g\left(\mu,\varphi\left(y^*\right),n\right)|_{y^*=0}=1$. □

Appendix D: Proof of Proposition 2

First consider the claim about μ. Differentiating both sides of (6) w.r.t. μ yields

$$\frac{\partial\left(LHS\right)}{\partial\mu}+\frac{\partial\left(LHS\right)}{\partial y^*}\frac{\partial y^*}{\partial\mu}=-\frac{\varphi''\left(y^*\right)}{\left[\varphi'\left(y^*\right)\right]^2}\frac{\partial y^*}{\partial\mu},$$

where LHS is short for the left-hand side of (6). From Appendix C we have that $\partial (LHS) / \partial y^*$ is strictly negative. Moreover, the same must be true for $\partial (LHS) / \partial \mu$ (since $\varphi (y^*)$ and μ have symmetric roles in the left-hand side of (6), and $\varphi' (y^*) > 0$). In addition, $\varphi'' (y^*)$ is negative. It follows that $\partial y^* / \partial \mu < 0$.

The claim about π_H is very straightforward given the above arguments and the proof is therefore omitted. □

Appendix E: Proof of Lemmas 1 and 2

Proof of Lemma 1 Suppose per contra that $\lim_{n\to\infty} y^* = y' > 0$, so that $\varphi(y') \in (0, 1)$ and $\varphi'(y') > 0$. Then the limit of the right-hand side of (6) as n goes to infinity equals $1/\varphi'(y')$, which by assumption is strictly positive. The limit of the left-hand side, however, equals zero. □

Lemma A1 *As $n \to \infty$, $\varphi (y^*) \, n \to \infty$.*

Proof First note that, by Lemma 1, the right-hand side of (6) goes to zero as $n \to \infty$. If Lemma A1 does not hold, then either (i) $\varphi (y^*)$ goes to zero at the same speed as $1/n$ or (ii) $\varphi (y^*)$ goes to zero faster than $1/n$. In either case, the numerator of the left-hand side of (6) must go to zero as $n \to \infty$, for otherwise the equality would not hold in the limit. In case (i), however, we can write

$$\lim_{n\to\infty} \left(1 - \mu\varphi\left(y^*\right)\right)^{n-1} = \lim_{n\to\infty} \left(1 - \frac{\mu}{n}\right)^{n-1},$$

which can be shown to equal $\mathrm{e}^{-\mu}$ (> 0). In case (ii), both the numerator and the denominator of the left-hand side of (6) go to zero, so we can use l'Hopital's rule (once) and then simplify:

$$\begin{aligned}
&\lim_{n\to\infty} \frac{1 - (1 - \mu\varphi (y^*))^{n-1}}{\mu\varphi (y^*) \, n} \\
&= \lim_{n\to\infty} \frac{-(1 - \mu\varphi (y^*))^{n-1} \left[\log (1 - \mu\varphi (y^*)) - \frac{\mu(n-1)}{1-\mu\varphi(y^*)} \varphi' (y^*) \frac{\partial y^*}{\partial n}\right]}{\mu \left(\varphi (y^*) + n\varphi' (y^*) \frac{\partial y^*}{\partial n}\right)} \\
&= \frac{\lim_{n\to\infty} (1 - \mu\varphi (y^*))^{n-1} \left[\lim_{n\to\infty} \left(\frac{\mu(n-1)}{1-\mu\varphi(y^*)}\right) \lim_{n\to\infty} \left(\varphi' (y^*) \frac{\partial y^*}{\partial n}\right) - \lim_{n\to\infty} \left[\log (1 - \mu\varphi (y^*))\right]\right]}{\mu \left(\lim_{n\to\infty} \varphi (y^*) + \lim_{n\to\infty} n \lim_{n\to\infty} \varphi' (y^*) \frac{\partial y^*}{\partial n}\right)}.
\end{aligned}$$

As the numerator of the left-hand side of (6) goes to zero, $\lim_{n\to\infty} (1 - \mu\varphi(y^*))^{n-1} = 1$. Moreover, by Lemma 1, $\lim_{n\to\infty} \varphi (y^*) = \lim_{n\to\infty} \left[\log (1 - \mu\varphi (y^*))\right] = 0$. The previous expression therefore simplifies to

$$\frac{\lim_{n\to\infty} \left(\frac{\mu(n-1)}{1-\mu\varphi(y^*)}\right) \lim_{n\to\infty} \varphi' (y^*) \frac{\partial y^*}{\partial n}}{\mu \lim_{n\to\infty} n \lim_{n\to\infty} \varphi' (y^*) \frac{\partial y^*}{\partial n}} = \frac{\lim_{n\to\infty} \left(\frac{n-1}{1-\mu\varphi(y^*)}\right)}{\lim_{n\to\infty} n} = 1.$$

This is strictly positive, whereas the limit of the right-hand side of (6) is zero. □

We can now prove Lemma 2. By Lemma A1, $\varphi(y^*)$ goes to zero slower than $1/n$. Hence, for n large enough, $\varphi(y^*) > 1/n$, which implies $\varphi(y^*) \geq 1/n^\beta$ for some $\beta \in (0,1)$. This means that, for such large n's,

$$\left(1 - \mu\varphi\left(y^*\right)\right)^{n-1} \leq \left(1 - \frac{\mu}{n^\beta}\right)^{n-1}.$$

One can show that as $n \to \infty$, the right-hand side of this inequality goes to zero; therefore, so does the left-hand side. □

Acknowledgements For helpful comments I thank an anonymous referee, Paul Heidhues, Anthony Heyes, Alexander Koch, Michael Mandler, Philipp Rehm, participants at the WZB conference “Interest Groups, Policy and Economic Performance” (Berlin, 2005) and at EEA 2006 (Vienna), and seminar participants at Royal Holloway.

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