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Insisting on a non-negative price: Oligopoly, uncertainty, welfare, and multiple equilibria

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Abstract

I study Cournot competition under incomplete, but symmetric, information about the intercept of the linear demand function, while assuming that market price must be non-negative for all demand realizations. Although the non-negativity assumption is very natural, it has only rarely been made in the earlier literature. Yet it has important economic consequences: (1) expected demand effectively becomes convex, which means that multiple (symmetric, pure strategy) equilibria can exist; and (2) expected total surplus can be larger when the firms do not know demand than when they do. The arguments of the paper are relevant also for price competition and for uncertainty about, e.g., cost or the number of firms, and these issues are discussed.

JEL classification: D42; D43; D80; L12; L13; L40

Keywords: Non-negativity constraint; Multiple equilibria; Value of information; Cournot competition; Antitrust policy

1. Introduction

Firms that are active in oligopolistic markets often face a considerable amount of uncertainty about demand, competitors' costs, and other market features that are important for the firms' decisions. Reflecting this fact, a large theoretical literature has developed that studies firm behavior under such uncertainty. In particular, a significant number of papers have investigated the firms' incentives to engage in information sharing, information acquisition, and strategic experimentation.[1]

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[1] For example, papers on information sharing include Clarke (1983), Gal-Or (1985, 1986), Li (1985), Novshek and Sonnenschein (1982), Ponssard (1979), Raith (1996), Sakai and Yamato (1989), Shapiro (1986), and Vives (1984, 1990).

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Although this is a rich literature with many important and useful insights, most of its contributions share an unappealing feature: they make assumptions that imply that either (for those models where firms choose quantities) market price can be negative or (for those models where firms choose prices) firms' output can be negative. The typical justification for making these assumptions is analytical tractability. It is also often argued that, by making appropriate additional assumptions about the distribution of the stochastic variable (e.g., by letting its variance be sufficiently low), one can ensure that a negative price/quantity will occur only with a low probability.[2] One problem with this argument, however, is that the real world situations that the models are supposed to capture often involve a substantial amount of uncertainty. One may therefore wonder whether the practice of using models where prices/quantities can be negative makes us overlook valuable insights.[3]

The first ones to investigate this question were Malueg and Tsutsui (1998a).[4] They study a Cournot duopoly model of information sharing and explicitly impose the constraint that market price must be non-negative for all demand realizations. Following the standard literature, Malueg and Tsutsui assume that the inverse demand function is linear and that the uncertainty concerns the stochastic intercept of this function. At an ex ante stage, each duopolist can commit to adding the private signal they will observe to a common pool. After having observed their private signals and possibly shared this information, the firms compete in quantities. In order to make the model sufficiently tractable and still be able to account for the non-negativity constraint, Malueg and Tsutsui assume that the stochastic intercept can take on only two distinct values. In spite of this, the algebra becomes quite involved. Nevertheless, by means of numerical examples, Malueg and Tsutsui show two results that were not previously known: when the non-negativity constraint is accounted for, information sharing can (i) be profitable for the firms and (ii) reduce social welfare (defined as the expected total surplus).

Neither (i) nor (ii) can occur in the standard model described in footnote 3, which assumed that cost, demand and conditional expectation functions are linear, and that there is no non-negativity constraint.[5] In particular, in that model, if the firms share their information, this has an unambiguously positive effect on expected total surplus: output increases if the additional information the firms get access to suggests that demand is relatively high, and otherwise it drops.

[2] See, for example, Vives (1984, p. 77, n. 2, 1999, Ch. 8, n. 6).

[3] A common modeling framework is to assume a linear cost function and a linear inverse demand function, $P(X)=a-bX$, where a is stochastic. Each firm observes a private signal s_i, and the joint distribution of a and s_i has the proporty that the conditional expectation function, $E(a|s_i)$, is linear. An example of such a distribution, which is often explicitly assumed, is a bivariate normal. If the demand intercept a indeed is normally distributed, then obviously market price will be negative for some realizations, since then a itself can be negative. But also if the distribution is such that a always takes non-negative values, market price will be negative if industry output (which is an endogenous variable) is large enough. See also the discussion in Malueg and Tsutsui (1998a, p. 364).

[4] The same authors have also investigated questions that are closely related to this one. Indeed, imposing a binding non-negativity constraint is only one way of deviating from the standard framework with linear conditional expectation functions that was discussed in footnote 3; Malueg and Tsutsui (1996, 1998b) study two of these. Malueg and Tsutsui (1996) assume uncertainty about the slope (rather than the intercept) of the demand function, and they show that this gives rise to result (i) mentioned below. Malueg and Tsutsui (1998b) show the same result in a model with uncertainty about the intercept but with alternative distributional assumptions regarding this stochastic variable.

[5] It is important to note, however, that imposing a non-negativity constraint is not *necessary* to obtain result (i) — this follows from Malueg and Tsutsui (1996, 1998b), who show the result without a binding non-negativity constraint but with other deviations from the standard model. Similarly, we should not expect a binding non-negativity constraint to be a necessary requirement for result (ii). Although I cannot offer a proof of this, it seems plausible that the result is driven by the convexity of the (expected) demand function (I am grateful to an anonymous referee and to Patrick Legros for suggesting this to me). The non-negativity constraint is just one way of obtaining the convexity.

Expected output is unaffected, however; moreover, the welfare gains in a good state more than compensates for the losses in a bad state. In Malueg and Tsutsui's example, in contrast, the fact that there is a non-negativity constraint on price that is sometimes binding makes expected output fall as a result of information sharing, which reduces social welfare.

The logic of the welfare result in Malueg and Tsutsui (1998a) suggests that it may not be driven by the sharing of private information per se, but rather by the fact that the firms, more generally, get access to more information. If this is correct, we may be able to learn more about the welfare effects of information sharing in an environment where there is a non-negativity constraint on price by studying a version of Malueg and Tsutsui's model in which information is symmetric (but still incomplete). In this paper I carry out such a study. I analyze the effects of providing all firms in an *n*-firm Cournot oligopoly with the same additional information. My model is thus, in one dimension, simpler than (or at least different from) Malueg and Tsutsui's in that it does not allow for any asymmetric information. In another dimension, however, it is richer than theirs as it allows for an arbitrary number of firms and one can relatively easily solve for its equilibria (for the whole parameter space, not only for some particular numerical examples). This modeling approach makes it possible to derive further insights into what the circumstances under which better informed firms can be bad for welfare look like, in particular in terms of the cost parameter and the number of firms in the industry. My model also, I believe, facilitates an understanding of the intuition of the results.[6]

The analysis shows that, like in Malueg and Tsutsui (1998a), taking the non-negativity constraint on market price into account can indeed reverse the welfare result found in the standard literature. The reason why informed firms can be detrimental to expected total surplus is that if ununiformed firm chooses a relatively large quantity and demand turns out to be low, its losses will, because of the non-negativity constraint on price, be limited to its production costs. Relative to a model in which price can be negative, this makes the firm bolder (or more aggressive) when choosing its output: it chooses a larger quantity than it would have done without the non-negativity constraint, which is good for the consumers and for total surplus. This "boldness effect" is particularly strong for low values of the marginal cost parameter, since then the overall production costs are low.[7]

The analysis also shows – again in contrast to the literature that ignores the non-negativity constraint – that taking the non-negativity constraint on market price into account can give rise to a multiplicity of equilibria (a result that does not appear in any of Malueg and Tsutsui's papers). The reason why this model gives rise to a multiplicity of equilibria is that the uncertainty about (inverse) demand together with the assumption that market price cannot be negative make the

[6] The important and special feature of Malueg and Tsutsui's model that market price must be non-negative for all demand realization is retained in my model. That is, if the firms have been optimistic about the demand to such an extent that a negative price is required for the market to clear, then market price simply equals zero. The fact that market price on these occasions is zero should not be interpreted too literally, however. A richer model, which I conjecture would give rise to qualitatively the same results as here, could assume that there are (constant unit) costs associated with selling the good. If market price falls below this cost level, the firms will prefer not to sell. In such a model, the non-negativity constraint assumed in the present paper would refer to the market price *net of selling costs*, and it could thus be binding also for a strictly positive (gross) market price. Another way of thinking about the non-negativity constraint would be that it refers to the market price net of marginal cost, and that there is a regulatory rule that makes a negative such net price illegal (justified by concerns for limit pricing).

[7] This is consistent with the analysis in Malueg and Tsutsui (1998a). They show their welfare result in an example in which the (constant) marginal cost equals zero. In the appendix of their paper, they also show the result in an example with a strictly positive (but low) marginal cost.

expected (inverse) demand function convex. It is well known that a Cournot model with known demand may have multiple equilibria if the demand function is sufficiently convex. Intuitively, for a demand function that is convex enough, the choice variables (i.e., the output levels) of a typical firm and one of its competitors are strategic complements: the marginal profit of a typical firm increases with the output of its competitor. As a result, multiple equilibria can be sustained through self-fulfilling beliefs on the part of the firms. In the model studied in the present paper, where demand is known to be linear (for positive price levels) but has an unknown intercept, a convexity of the expected demand schedule arises naturally because of the non-negativity constraint on price, and this creates a multiplicity of equilibria for the same reason as in a model with known and sufficiently convex demand.[8]

Although the arguments of the paper are developed in a Cournot setting and with uncertainty about the demand intercept, the points I make are relevant also under other assumptions. Later in the paper I will discuss the effects of imposing a non-negativity constraint on quantities in the Bertrand model (see the end of Section 3.2) and the consequences of assuming uncertainty about other parameters than the demand intercept (see the concluding section of Section 4).

The remainder of the paper is organized as follows. In Section 2 the model is described. Section 3 presents the analysis and the results: Section 3.1 demonstrates and discusses the multiplicity result and Section 3.2 does the same with the welfare result. Section 4 concludes. Most of the algebra is relegated to an appendix.

2. Model

Consider a Cournot model with $n \geq 1$ firms producing a homogeneous good. The firms are identical and indexed by $i \in \{1, 2,\ldots, n\}$. Each one of them faces a linear inverse demand function $p(X) = \max\{0, \alpha - X\}$ where p is price, $X \equiv \sum_{i=1}^{n} x_i$ is industry output, x_i is the firm i's output, and $\alpha > 0$ is an exogenous parameter. All firms have the same constant marginal cost technology, with marginal cost denoted $c > 0$, and there is no fixed cost.

The intercept of the demand function, α, is unknown by the firms. The intercept is either "low," in which case $\alpha = a - \Delta$, or "high," in which case $\alpha = a + \Delta$, with $a > \Delta > 0$ and $a > c$. Each one of the states of nature occurs with equal probability: $\Pr(\alpha = a - \Delta) = \Pr(\alpha = a + \Delta) = 1/2$.

Each firm i is risk neutral and maximizes its expected profits. Its choice variable is its own output, $x_i \geq 0$, which it chooses simultaneously with the other firms. I will confine attention to pure strategy Nash equilibria of this game.

3. Analysis and results

The algebra of the model is worked out in Appendix A. Here I will just state the results and subsequently explain the logic behind them. First, however, we need some more terminology and notation.

[8] In a complete information Cournot model more generally (also in symmetric versions of this model), there can exist multiple equilibria of another kind, namely equilibria in which one firm or a subset of firms produce a positive quantity whereas the others are inactive, producing nothing; see Amir and Lambson (2000). Such equilibria will not exist, however, in the model that I investigate. The standard formulation of the linear–quadratic Cournot model with incomplete information – which allows for negative prices and quantities – does have a unique equilibrium. This is typically proven by rewriting (using a technique suggested by Basar and Ho, 1974) a firm's payoff function in a way that does not alter the first-order condition but which transforms the problem into a team decision problem. Then a uniqueness theorem due to Radner (1962) can be used. See, for example, Vives (1999).

Let us make the observation that the fact that the intercept of the inverse demand function is stochastic together with the non-negativity constraint on market price imply that the *expected* price schedule, $E\{p(X)\}$, has a kink. (To see this, the reader may want to draw a figure.) The kink is located at that level of industry output where the price schedule in a low-demand state meets the horizontal axis,[9] $a-\Delta \equiv X^{\text{kink}}$. I will say that if $X < X^{\text{kink}}$, then industry output is located left of the kink; and if $X^{\text{kink}} < X$, then industry output is located right of the kink.

Let the cut-off values Δ^* and Δ^{**} be defined by

$$\Delta^* \equiv \frac{(2-\sqrt{2})a + (n+2\sqrt{2}-3)c}{n+\sqrt{2}-1}, \tag{1}$$

$$\Delta^{**} \equiv \frac{2(\sqrt{2}-1)a + (n+3-2\sqrt{2})c}{n+1}. \tag{2}$$

One can readily verify that $0 < \Delta^* < \Delta^{**} < a$ for $n \geq 2$, and $0 < \Delta^* = \Delta^{**} < a$ for $n=1$. Moreover, let the output levels x_L^* and x_R^* be defined by

$$x_L^* \equiv \frac{a-c}{n+1}, \quad x_R^* \equiv \frac{a+\Delta-2c}{n+1}. \tag{3}$$

Proposition 1.

- *For $\Delta \in (0, \Delta^*)$ there is a unique pure strategy equilibrium. In this equilibrium, each firm produces x_L^*, and industry output is located left of the kink.*
- *For $\Delta \in (\Delta^{**}, a)$ there is a unique pure strategy equilibrium. In this equilibrium, each firm's output equals x_R^*, and industry output is located right of the kink.*
- *For $\Delta \in [\Delta^*, \Delta^{**}]$ there are exactly two pure strategy equilibria. One is left of the kink with each firm's output equal to x_L^*, whereas the other is right of the kink with each firm's output equal to x_R^*.*

3.1. Multiplicity of equilibria

Fig. 1 depicts Δ^* and Δ^{**} as (linear) functions of c for the case where $n \geq 2$ (recall that Δ^* and Δ^{**} coincide if $n=1$). We know from Proposition 1 that an equilibrium in which industry output is right of the kink exists above the graph of Δ^*, and an equilibrium in which industry output is left of the kink exists beneath the graph of Δ^{**}. Thus, in the region between the two graphs (the shadowed area in the figure) an equilibrium left of the kink coexists with an equilibrium right of the kink.

Clearly, the reason why this model gives rise to a multiplicity of equilibria is related to the non-negativity constraint and the kink that it implies. The crucial model feature, however, is not the existence of a kink per se, but the fact that the expected price schedule is convex in a region where it pays off for the firms to produce. It is well known from work on the Cournot model under complete information that there can exist multiple equilibria if the demand function is sufficiently

[9] Of course, the expected price schedule has a kink also at the point where it meets the horizontal axis. Throughout the paper, however, the word "kink" will refer to the kink on the downward sloping part of the expected price schedule.

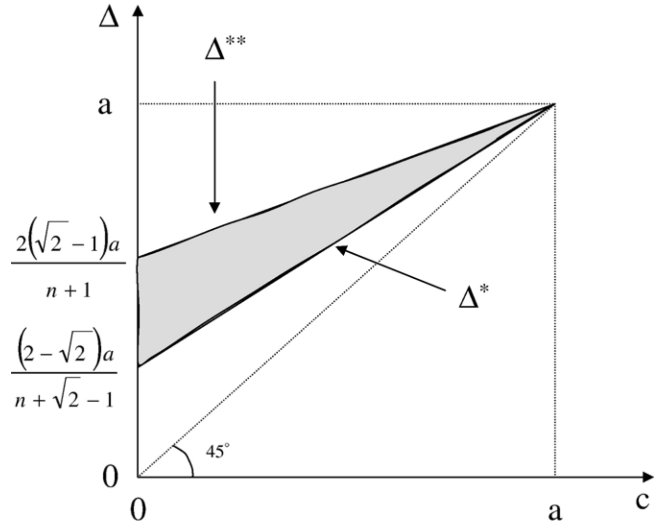

Fig. 1. An equilibrium left of the kink exists beneath Δ**, and an equilibrium right of the kink exists above Δ*. Hence, in the shadowed area between Δ* and Δ** both equilibria exist.

convex. The reason why a convexity of demand has this effect is that it tends to create a strategic complementarity between a firm's own output and its competitors' output: the marginal gain in profits from increasing the own strategic variable is increasing in each of the competitors' strategic variables.[10] As a consequence, a low-output equilibrium can exist simultaneously with a high-output equilibrium, since beliefs about the competitors' behavior become self-fulfilling.

In the model studied here, the requirement that price cannot be negative creates a convexity of the *expected* demand schedule, which again leads to a strategic complementarity and thus the possibility of multiple equilibria. In particular, although otherwise linear and downward sloping, firm *i*'s best-response correspondence makes, because of the kink, one jump upwards. The jump occurs at an output level of the other firms that is just large enough to make it optimal for firm *i* to produce such a large quantity itself that industry output locates right of the kink instead of left of it.

Fig. 1 also tells us that as the number of firms in the market, *n*, increases, the intercept of the graphs of Δ* and Δ** move downwards and, in the limit, both straight lines approach the 45° line. Hence, as the market approaches perfect competition, the scope for multiple equilibria in this model vanishes.

It is important to reiterate that the fact that a convexity of the demand function can lead to a multiplicity of equilibria is well understood in the literature. The contributions here are: (i) to point out that, in a model with otherwise linear demand, one possible source of such a convexity is a non-negativity constraint on price; and (ii) to show (see Fig. 1 and Proposition 1) that the non-negativity constraint can indeed give rise to two coexisting equilibria.[11]

[10] To see this, suppose inverse demand is known and given by $D(x+y)$, where x is own output and y is the competitors' joint output, and denote the cost function by $C(x)$. Then own profits are given by $\pi(x, y)=D(x+y)x-C(x)$. Differentiating π twice, first with respect to x and then with respect to y, one has $\pi_{12}(x, y)=D'(x+y)+xD''(x+y)$. This expression can be positive if $D''(x+y)$ is positive and sufficiently large, even if the demand function is downward sloping.

[11] In Lagerlöf (in press) I study a continuous-state version of the present model and investigate what families of distribution functions of the stochastic demand intercept that yield a unique equilibrium.

3.2. Welfare

Let us now ask the question how the fact that the firms have incomplete information affects expected profits and expected total surplus. The model that I will use as a benchmark for comparison is identical to the one described in Section 2, except that in the benchmark all firms know the realization of the demand shock when they make their output decisions. I will make the comparison from an ex ante perspective.

When the value of the demand intercept, $\alpha \in \{a-\Delta,\ a+\Delta\}$, is common knowledge, we know from standard calculations that there is a unique equilibrium in which each firm produces $x_B \equiv \max\ \{0, (\alpha - c)/(n+1)\}$ (the subscript B is short for "benchmark"). Thus, in a high-demand state the output level x_B is always strictly positive, whereas in a low-demand state $x_B=0$ for $\Delta \le a-c$. Denote by $\pi_B(\alpha)$ and $\mathrm{CS}_B(\alpha)$ a firm's profits respectively the consumer surplus in the benchmark model, given a realization of the demand intercept α.

We have

$$\pi_B(\alpha) = (\alpha - c - nx_B)x_B, \quad \mathrm{CS}_B(\alpha) = \frac{(nx_B)^2}{2}. \tag{4}$$

Total surplus (or "welfare") in the benchmark model, given a realization of α, then equals $W_B(\alpha) = \mathrm{CS}_B(\alpha) + n\pi_B(\alpha)$. The expected profits and expected total surplus, $E\pi_B$ and EW_B, are defined as the expected values of $\pi_B(\alpha)$ and $W_B(\alpha)$, given that the probability of each state is 1/2.

Now return to the incomplete information model. In an equilibrium left of the kink, the non-negativity constraint on price is never binding. Hence, results that are novel relative to the existing literature can be expected to be found only in an equilibrium right of the kink. Therefore, I will make the profit and total surplus comparison only for such an equilibrium of the incomplete information model.

Accordingly, assume that $\Delta \in (\Delta^*, a)$ and that an equilibrium right of the kink is played. Denote by $\pi^*(\alpha)$ and $\mathrm{CS}^*(\alpha)$ a firm's profits and the consumer surplus, respectively, in the incomplete information model, given a realization of the demand intercept α. In a low-demand state, market price is zero. Hence, $\pi^*(a-\Delta) = -cx_R^*$ (i.e., the firm has no revenues, so its profits equal minus its production costs). In a high-demand state, $\pi^*(\alpha)$ is defined analogously to $\pi_B(\alpha)$ in (4), but with x_R^* substituted for x_B. Similarly, since market price is zero in a low-demand state, I say that $\mathrm{CS}^*(a-\Delta)$ is given by the whole area beneath the demand schedule, $\mathrm{CS}^*(a-\Delta) = (a-\Delta)^2/2$ (i.e., all the goods that are produced are handed over to the consumers free of charge). In a high-demand state, $\mathrm{CS}^*(\alpha)$ is defined analogously to $\mathrm{CS}_B(\alpha)$ in (4), but with R substituted for x_B. Total surplus, given a realization of α, is defined by $W^*(\alpha) = \mathrm{CS}^*(\alpha) + n\pi^*(\alpha)$. Finally, the expected profit and expected total surplus, $E\pi^*$ and EW^*, are defined as the expected values of $\pi^*(\alpha)$ and $W^*(\alpha)$, given that the probability of each state is 1/2.

The following result is proven in Appendix A.

Proposition 2. *Suppose $\Delta \in (\Delta^*, a)$ and that an equilibrium right of the kink is played in the incomplete information model. Then:*

a) *Expected profits are always strictly higher under complete information than under incomplete information (i.e., $E\pi^* > E\pi_B$).*

b) *Expected total surplus under incomplete information is strictly higher than the expected social surplus under complete information (i.e., $EW^* > EW_B$) if and only if $\Delta < \varphi$ (a, c, n), where*

$$\varphi(a, c, n) \equiv a - \frac{2a - c}{1 + \sqrt{1 + \frac{2a-c}{2n(n+2)c}}}. \tag{5}$$

Part a) of Proposition 2 is quite intuitive and in line with what we know from the existing literature. Part b) is illustrated in Fig. 2. This figure shows (the relevant part of) the graph of the function φ, in the same (Δ, c) space as in Fig. 1. In the (non-empty) subset of the parameter space beneath this graph and above the graph of Δ^* (the shadowed area in the figure), an equilibrium right of the kink exists in the model with incomplete information and expected total surplus in that equilibrium is higher than in the benchmark model where the firms do know demand.

The reason why expected total surplus can be lower when the firms are informed is that a firm that does not know demand is bolder (or more aggressive) when choosing its output: it chooses a quantity that is large relative to what it would have chosen on average if it had known demand. The reason for this, in turn, is that if the firm chooses a relatively large quantity and demand turns out to be low, its losses will, because of the non-negativity constraint on price, be limited to its production costs.

Given this logic, we should expect the "boldness effect" to be stronger the lower is the marginal/average cost. Indeed, provided an equilibrium right of the kink exists, $\partial\varphi / \partial c < 0$. Moreover, $\lim_{c \to 0} \varphi(a, c, n) = a$, which means that in the limit, as the constant marginal cost approaches zero, informed firms are detrimental to expected total surplus for all $\Delta \in (\Delta^*, a)$, i.e., whenever an equilibrium right of the kink exists. We also have that $\partial\varphi / \partial n < 0$: stiffer competition decreases the cut-off value below which Δ must be for informed firms to be bad for expected total surplus. Indeed, in the limit as n approaches infinity, informed firms are never detrimental to expected total surplus.

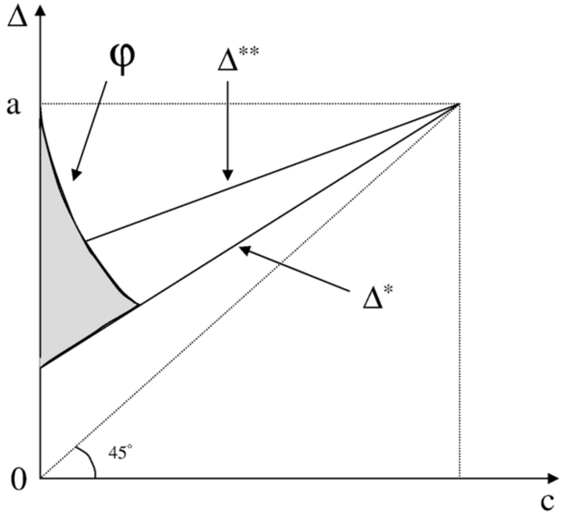

Fig. 2. In the shadowed region is, given that an equilibrium right of the kink is played, expected total surplus lower when the firms know demand than when they do not.

To see the significance of Proposition 2b, let us briefly review what received theory has to say about the welfare effects of better informed firms in an oligopoly (or monopoly) market. Vives (1984) studies a linear-quadratic differentiated goods duopoly model with uncertainty about the intercept of the demand function. In one version of his model he assumes Cournot competition, whereas in another there is Bertrand competition. Vives does not impose any non-negativity constraint on the variable that is not chosen by the firms (i.e., in the Cournot model price can be negative and in the Bertrand model output can be negative). He computes the social value (i.e., the difference in expected total surplus) of the firms' having access to more information and shows that there is a strict dichotomy between the Cournot and Bertrand models: the social value of information is positive under Cournot and negative under Bertrand competition.[12]

How can we understand this dichotomy? One might have expected more information to be socially beneficial under both Cournot and Bertrand competition, since it should help the firms to tailor their decisions to actual demand, thus facilitating the exploitation of gains from trade and making the pie to be shared between the firms and the consumers bigger. Of course, however, the firms do not care about the size of this pie per se but about their share of it. Still, it turns out that when the firms are quantity setters, then a firm's objective of maximizing the share of the pie is relatively well aligned with the social goal of maximizing the pie size; for price setters, though, these goals are less well aligned. This difference between quantity setting and price setting is due to the facts that: (i) socially "good behavior" on the part of the firms (i.e., their choosing large quantities respectively low prices) is more valuable in a high-demand state than in a low-demand state (basically because the traded quantity is larger in a high-demand state); and (ii) a quantity setter who gets access to information responds by producing more in a high-demand state and less in a low-demand state, whereas a price setter responds by choosing a high price in a high-demand state and a low price in a low-demand state.[13]

The effect discussed in the previous paragraph is present also in the model studied in the current paper — indeed, this is why expected total surplus is greater with informed than with uninformed firms whenever $\Delta > \varphi(a, c, n)$. In the model studied here, however, there is a non-negativity constraint on price, which gives rise to the boldness effect discussed earlier. As a result, whether information is good or bad does not depend only on whether the firms choose price or quantity, but also on the strength of the boldness effect. In particular, when the marginal cost parameter is relatively low, which makes the boldness effect strong, information is bad also under Cournot competition.

Although the case with Bertrand competition (with differentiated goods) is not analyzed in the present paper, it is fairly straightforward to understand how the logic would work in such a model: a non-negativity constraint on quantity would make it more tempting for firms to choose a high price. Thus, in a model where firms choose prices and there is a non-negativity constraint on quantity, the boldness effect would tend to make uninformed firms harmful to consumers, since it creates an incentive to set a higher price. Also in such a model, the boldness effect would be stronger when the production costs are relatively low. Hence, one should expect that for sufficiently low values of the marginal cost parameter, the boldness effect would be stronger than the effect present in Vives (1984) also under Bertrand competition, with the

[12] For discussions of the welfare effects of information sharing, see also, for example, Clarke (1983), Novshek and Sonnenschein (1982), Sakai and Yamato (1989), Shapiro (1986), and Vives (1990).

[13] Kühn and Vives (1995) discuss this intuition at length and also illustrate it in figures. See also Weitzman (1974) for an early analysis of the difference between price and quantity setting under uncertainty.

result being that, for a subset of the parameter space, informed firms is good for expected total surplus.

In summary, the strict dichotomy between Cournot and Bertrand is broken once we introduce the non-negativity constraint: depending on the parameters, information can be either good or bad in each one of the two models. In particular, the traditional welfare result in the Cournot model (information is good) is reversed for sufficiently low values of the marginal cost parameter (for then the boldness effect is relatively strong), and we should expect the analogous result in the Bertrand model.

4. Concluding remarks

Ignoring the non-negativity constraint on price when modeling uncertainty in oligopolistic (or monopolistic) markets may make us overlook important economic insights. In particular, taking the non-negativity constraint explicitly into account in a simple Cournot model with demand uncertainty can lead to (i) a multiplicity of equilibria and to (ii) the phenomenon that expected total surplus is larger when the firms do not know the demand than when they do. Observation (ii) was first made by Malueg and Tsutsui (1998a), by means of a numerical duopoly example with private information. In the present paper, thanks to the fact that here information is symmetric (but incomplete), I obtained a much more tractable model and I could therefore derive some further results. These suggest that the tendency for informed firms to be bad for welfare is stronger when marginal/average cost is low and that providing the firms with more information is always good if the number of firms in the market is sufficiently large.

The arguments of the paper are relevant also for price competition (see the end of Section 3.2) and for uncertainty about market features other than the demand intercept. Suppose, for example, that there are at least two firms in a Cournot market and that each firm has private information about its own (constant) marginal cost, thus making its output decision contingent on this information. Then, from the point of view of an individual firm, aggregate output will be stochastic and the non-negativity constraint on price will, at least for some possible output choices, be binding with positive probability. In such a model the non-negativity constraint should play a role that is very similar to the one explored in the model of the present paper. The same is true for anything else that is private information to a firm and which affects its output choice. It could also be that the number of firms in the market is unknown to an individual firm, which again would make aggregate output stochastic from the point of view of that firm. Examining these and other alternative models in greater detail could yield further insights.

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Appendix A

Proof of Proposition 1. Let us first look for equilibria left of the kink, i.e., where $X^* < X^{\text{kink}}$ $(\equiv a-\Delta)$. It is straightforward to verify that in such an equilibrium, given that it exists, all firms produce the same quantity, namely x_{L}^* as defined in (3). Now consider firm i's incentive to deviate to some x_i such that $x_i+(n-1)\,x_{\text{L}}^* \geq a-\Delta$ or, rewriting, to some x_i such that

$$x_i \geq a-\Delta-(n-1)x_{\text{L}}^* = \frac{2a-(n+1)\Delta+(n-1)c}{n+1} \equiv A(\Delta).$$

If deviating to such an x_i, firm i's expected profits are given by

$$\left(\frac{a+\Delta-x_i-(n-1)x_{\text{L}}^*}{2}-c\right)x_i = \left(\frac{2a+(n+1)\Delta-(n+3)c}{2(n+1)}-\frac{1}{2}x_i\right)x_i. \tag{6}$$

Maximizing this expression with respect to x_i subject to $x_i \geq A\,(\Delta)$ yields an optimal $\hat{x}$ given by

$$\hat{x} = \begin{cases} A(\Delta) & \text{if } \Delta \leq \dfrac{2a+(3n+1)c}{3(n+1)} \\ \dfrac{2a+(n+1)\Delta-(n+3)c}{2(n+1)} & \text{otherwise.} \end{cases} \tag{7}$$

Clearly, when Δ is such that the constraint $x_i \geq A\,(\Delta)$ is binding, then firm i does not have an incentive to deviate. Consider the case where the constraint is not binding. Compute firm i's expected profit if not deviating and if deviating:

$$\pi_i(\text{not deviating}) = \frac{(a-c)^2}{(n+1)^2},$$

$$\pi_i(\text{deviating}) = \left(\hat{x}-\frac{1}{2}\hat{x}\right)\hat{x} = \frac{[2a+(n+1)\Delta-(n+3)c]^2}{8(n+1)^2}$$

(the last two equalities make use of (6) and (7)). It is a straightforward exercise to verify that π_i (deviating) is monotone increasing in Δ for all $\Delta > [2a+(3n+1)\,c]/3\,(n+1)$. Moreover, aggregate output in an equilibrium left of the kink equals nx_{L}^*, so we will indeed have an equilibrium left of the kink if and only if $nx_{\text{L}}^* < X^{\text{kink}}$ or, rewriting, $\Delta < (a+nc)/(n+1)$. Evaluating π_i (deviating) at $\Delta = (a+nc)/(n+1)$ yields

$$\pi_i(\text{deviating})|_{\Delta=\frac{a+nc}{n+1}} = \frac{9(a-c)^2}{8(n+1)^2} > \pi_i(\text{not deviating}).$$

Thus, there exists a unique Δ_A, satisfying

$$\frac{2a+(3n+1)c}{3(n+1)} < \Delta_A < \frac{a+nc}{n+1}$$

and

$$\frac{(a-c)^2}{(n+1)^2} = \frac{[2a+(n+1)\Delta_A-(n+3)c]^2}{8(n+1)^2}, \tag{8}$$

such that firm i will not have an incentive to deviate if and only if $\Delta \leq \Delta_A$. The equality in (8) has a unique root in the relevant interval which is given by $\Delta_A = \Delta^{**}$, where Δ^{**} is defined in (2).

Now let us look for equilibria right of the kink, i.e., where $X > X^{\text{kink}}$. It is straightforward to verify that in such an equilibrium, given that it exists, all firms produce the same quantity, namely x_{R}^* as defined in (3). This output is non-negative only if $\Delta \geq 2c - a$, which thus is a necessary condition for an equilibrium right of the kink to exist. Consider firm i's incentive to deviate to some x_i such that $x_i + (n-1)\, x_{\text{R}}^* \leq X^{\text{kink}}$ or, rewriting, to some x_i such that

$$x_i \leq a - \Delta - (n-1)x_{\text{R}}^* = \frac{2[a - n\Delta + (n-1)c]}{n+1} \equiv B(\Delta).$$

Note that $B(\Delta) < 0$ if $\Delta > [a + (n-1)\, c]/n$, in which case firm i is unable to move industry output left of the kink. Hence, suppose that Δ is small enough so that $B(\Delta) \geq 0$. Now, if deviating to an $x_i \in [0, B(\Delta)]$, firm i's expected profits are given by

$$[a - x_i - (n-1)x_{\text{R}}^* - c]x_i = \left(\frac{2a - (n-1)\Delta + (n-3)c}{n+1} - x_i\right)x_i. \tag{9}$$

Maximizing this expression with respect to x_i subject to $x_i \leq B(\Delta)$ (the constraint $x_i \geq 0$ will not be binding when $B(\Delta) \geq 0$) yields an optimal $\tilde{x}$ given by

$$\tilde{x} = \begin{cases} \dfrac{2a - (n-1)\Delta - (n-3)c}{2(n+1)} & \text{if } \Delta \leq \dfrac{2a + (3n-1)c}{3n+1} \\ B(\Delta) & \text{otherwise.} \end{cases} \tag{10}$$

Clearly, when Δ is such that the constraint $x_i \leq B(\Delta)$ is binding, then firm i does not have an incentive to deviate. Consider the case where the constraint is not binding. Calculate firm i's profit if not deviating and if deviating:

$$\pi_i^{\text{Not Dev}} = \frac{(a + \Delta - 2c)^2}{2(n+1)^2}, \qquad \pi_i^{\text{Dev}} = (2\tilde{x} - \tilde{x})\tilde{x} = \frac{[2a - (n-1)\Delta + (n-3)c]^2}{4(n+1)^2}$$

(the last two equalities make use of (9) and (10)). Given that $x_{\text{R}}^* \geq 0$ (so that $\Delta \geq 2c - a$), $\pi_i^{\text{Not Dev}}$ is monotone increasing in Δ, and one can verify that π_i^{Dev} is monotone decreasing in Δ provided that $B(\Delta) \geq 0$. Moreover, aggregate output in an equilibrium right of the kink equals nx_{R}^*, so we will indeed have an equilibrium right of the kink if $nx_{\text{R}}^* > X^{\text{kink}}$ or, rewriting, $\Delta > (a + 2nc)/(2n+1)$.

Evaluating the above two expressions at $\Delta = (a + 2nc)/(2n+1)$, we have

$$\pi_i^{\text{Not Dev}}\Big|_{\Delta = \frac{a+2nc}{2n+1}} = \frac{2(a-c)^2}{(2n+1)^2}, \qquad \pi_i^{\text{Dev}}\Big|_{\Delta = \frac{a+2nc}{2n+1}} = \frac{9(a-c)^2}{4(2n+1)^2}.$$

Hence, for $\Delta = (a - 2nc)/(2n+1)$, $\pi_i^{\text{Dev}} > \pi_i^{\text{Not Dev}}$. This means that there exists a unique Δ_B, satisfying

$$\frac{a + 2nc}{2n+1} < \Delta_B < \frac{2a + (3n-1)c}{3n+1}$$

and

$$\frac{(a + \Delta_B - 2c)^2}{2(n+1)^2} = \frac{[2a - (n-1)\Delta_B + (n-3)c]^2}{4(n+1)^2}, \tag{11}$$

such that firm i will not have an incentive to deviate if and only if $\Delta \geq \Delta_B$. The equality in (11) has a unique root in the relevant interval which is given by $\Delta_B = \Delta^*$, where Δ^* is defined in (1). □

Proof of Proposition 2. The proof makes use of Table A1, the entries of which can be calculated by using the definitions and formulas provided in Section 3.2. Let us first prove part a) of the proposition. For the case $\Delta > a - c$ it follows immediately from Table A1 that $E\pi_B > E\pi^*$. Next consider the case $\Delta \leq a - c$. From Table A1 we have that $E\pi_B > E\pi^* \Leftrightarrow 2\,[(a-c)^2 + \Delta^2] > (a + \Delta - 2c)^2$, which can be rewritten as

$$[\Delta - (a - 2c)]^2 > 2[(a - 2c)^2 - (a - c)^2] = -2c(2a - 3c).$$

Recall that $\Delta > \Delta^*$ implies $\Delta > c$. Thus, a necessary condition for having $\Delta \leq a - c$ is that $a > 2c$. But this means that the right-hand side of the inequality above is strictly negative, so the inequality must always hold.

Let us now prove b). First consider the case $\Delta > a - c$. Here, using Table A1 and simplifying, the inequality $\mathrm{EW}^* < \mathrm{EW}_B$ can be written as $\lambda\,(a, c, \Delta, n) > 0$, where $\lambda\,(a, c, \Delta, n) \equiv n\,(n+2)\,c\,[2$

Table A1
Profits, consumer surplus, and welfare under complete and incomplete information

	C.I.: $\Delta \leq (a-c)$	C.I.: $\Delta > (a-c)$	Incompl. Info.
$\pi\,(a - \Delta)$	$\frac{(a-\Delta-c)^2}{(n+1)^2}$	0	$-cx_R^*$
$\pi\,(a+\Delta)$	$\frac{(a+\Delta-c)^2}{(n+1)^2}$	$\frac{(a+\Delta-c)^2}{(n+1)^2}$	$\frac{[a+\Delta+(n-1)c](a+\Delta-2c)}{(n+1)^2}$
$E\pi$	$\frac{(a-c)^2+\Delta^2}{(n+1)^2}$	$\frac{(a+\Delta-c)^2}{2(n+1)^2}$	$\frac{(a+\Delta-2c)^2}{2(n+1)^2}$
CS $(a-\Delta)$	$\frac{n^2(a-\Delta-c)^2}{2(n+1)^2}$	0	$\frac{(a-\Delta)^2}{2}$
CS $(a+\Delta)$	$\frac{n^2(a+\Delta-c)^2}{2(n+1)^2}$	$\frac{n^2(a+\Delta-c)^2}{2(n+1)^2}$	$\frac{n^2(a+\Delta-2c)^2}{2(n+1)^2}$
ECS	$\frac{n^2[(a-c)^2+\Delta^2]}{2(n+1)^2}$	$\frac{n^2(a+\Delta-c)^2}{4(n+1)^2}$	$\frac{(n+1)^2(a-\Delta)^2+n^2(a+\Delta-2c)^2}{4(n+1)^2}$
EW	$\frac{n(n+2)[(a-c)^2+\Delta^2]}{2(n+1)^2}$	$\frac{n(n+2)(a+\Delta-c)^2}{4(n+1)^2}$	$\frac{(n+1)^2(a-\Delta)^2+n(n+2)(a+\Delta-2c)^2}{4(n+1)^2}$

Columns two and three concern the benchmark model with complete information, whereas the last column refers to the incomplete information model. The notation in the first column is for simplicity written without the subindex "B" or the superindex "*". Thus, $\pi\,(a-\Delta)$, for example, should be understood as either $\pi_B\,(a-\Delta)$ or $\pi^*\,(a-\Delta)$, depending on which column one is reading from.

$(a+\Delta)-3c]-(n+1)^2\,(a-\Delta)^2$. The function λ is strictly increasing in Δ. Recall that a necessary condition for an equilibrium right of the kink to exist is that $\Delta>c$. Hence, in order to show that $EW^*<EW_B$ for the relevant parameters (i.e., for all $\Delta>\max\{a-c, c\}$), it suffices to show that: for all $a\geq 2c$, $\lambda(a, c, \Delta, n)|_{\Delta=a-c}>0$; and for all $a\in(c, 2c)$, $\lambda(a, c, \Delta, n)|_{\Delta=c}>0$. We have $\lambda(a, c, \Delta, n)|_{\Delta=a-c}=4n(n+2)ac-(6n^2+12n+1)c^2$, which is indeed strictly positive for all $a\geq 2c$. We also have $\lambda(a, c, \Delta, n)|_{\Delta=c}=n(n+2)c(2a-c)-(n+1)^2(a-c)^2$, which is concave in a and strictly positive evaluated at $a=c$ and $a=2c$. Hence, $\lambda(a, c, \Delta, n)|_{\Delta=c}>0$ for all $a\in(c, 2c)$.

It remains to consider the case $\Delta\leq a-c$. Using Table A1 and simplifying, $EW_B<EW^*$ can be written as $n(n+2)\{2[(a-c)^2+\Delta^2]-(a+\Delta-2c)^2\}<(n+1)^2(a-\Delta)^2$. Rewriting again and then completing the square (with respect to Δ) yield

$$\{[a+2n(n+2)c]-\Delta\}^2>2n(n+2)c[(2n^2+4n-1)c+2a].$$

Since $a+2n(n+2)c>\Delta$ and the right-hand side is positive, we can take the square root of both sides of the above inequality. Doing this and rewriting yield

$$\begin{aligned}\Delta &< a+2n(n+2)c-\sqrt{2n(n+2)c[(2n^2+4n-1)c+2a]}\\ &= a+2n(n+2)c\left[1-\frac{\sqrt{(2n^2+4n-1)c+2a}}{\sqrt{2n(n+2)c}}\right]\\ &= a+2n(n+2)c\left[\frac{1-\frac{(2n^2+4n-1)c+2a}{2n(n+2)c}}{1+\sqrt{\frac{(2n^2+4n-1)c+2a}{2n(n+2)c}}}\right]\\ &= a+2n(n+2)c\left[\frac{\frac{2n(n+2)c-(2n^2+4n-1)c-2a}{2n(n+2)c}}{1+\sqrt{\frac{(2n^2+4n-1)c+2a}{2n(n+2)c}}}\right],\end{aligned}$$

which simplifies to $\Delta<\varphi(a, c, n)$. □

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