

# Supplementary Material to “Efficiency-Enhancing Signalling in the Samaritan’s Dilemma”

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## 1 Introduction

This note contains a proof that was left out from Lagerlöf (2003) due to space constraints. In particular, it shows that no pooling equilibrium of the model in that paper survives the intuitive criterion.

## 2 Pooling equilibria

Concerning the particular model considered in Lagerlöf (2003), the intuitive criterion (Cho and Kreps, 1987) says the following. Fix some equilibrium of the game. Suppose that, for any possible best response by  $A$ , the equilibrium utility for the low type of  $B$  is strictly greater than the utility the low type would receive if he saved some out-of-equilibrium amount  $s$ , regardless of  $A$ ’s beliefs. Moreover, suppose that this is not true for the high type; that is, there are some beliefs  $\tilde{\mu}(s)$  such that, if  $A$  makes a best response given these beliefs, the high type would get a higher payoff if choosing  $s$  instead of  $s_H^*$ . Then, if  $A$  observes the saving level  $s$ ,  $A$  should place zero probability on the possibility that  $B$  is the low type,  $\tilde{\mu}(s) = 1$ . This requirement must hold for all out-of-equilibrium saving levels  $s$ . If it does, the equilibrium is said to survive the intuitive criterion.

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**Proposition.** *In the model in Lagerlöf (2003), no pooling equilibrium survives the intuitive criterion.*

*Proof.* In a pooling equilibrium, both types make the same saving decision, say  $s_L^* = s_H^* = s^*$ . After having observed this saving level,  $A$  makes a transfer to  $B$  according to equation (3) in Lagerlöf (2003) but with  $E(\beta) \equiv \mu\beta_H + (1 - \mu)\beta_L$  substituted for  $\beta$ . In the figure below, the graph of this optimal transfer function is depicted as the intermediate straight line; the other two straight lines in the figure represent  $A$ 's optimal transfer if believing that  $\beta = \beta_L$  respectively  $\beta = \beta_H$ . Let  $\hat{s}$  be the saving level for which the low type's indifference curve through the point  $(s, t) = (0, \alpha\beta_L\omega / (1 + \alpha\beta_L))$ —call it  $I_L^0$ —crosses the intermediate transfer line. A necessary condition for a pooling equilibrium to exist is that  $s^* \leq \hat{s}$ ; for if we had  $s^* > \hat{s}$ , the low type could deviate profitably to  $s = 0$ . Suppose there is indeed a pooling equilibrium in which  $s^* \in [0, \hat{s}]$ . Let  $I_L^*$  (respectively,  $I_H^*$ ) denote the indifference curve for the low (respectively, high) type that crosses the intermediate transfer line at  $s = s^*$ . By the single-crossing property,  $I_H^*$  lies below  $I_L^*$  for all  $s > s^*$ . As a consequence, if we denote by  $y$  (respectively,  $y'$ ) the value of  $s$  for which  $I_L^*$  (respectively,  $I_H^*$ ) crosses the upper transfer line, we have  $y < y'$ . Clearly, for any beliefs  $\tilde{\mu}(y'')$ , the low type prefers his equilibrium payoff to the payoff he would get if deviating to some  $s = y'' \in (y, y')$ . The high type, on the other hand, would have an incentive to deviate to  $y''$  if he thereby were perceived as the high type. Hence,  $s^*$  can be part of an intuitive equilibrium only if it can be supported by beliefs assigning zero probability to the low type playing  $s = y''$ , that is,  $\tilde{\mu}(y'') = 1$ . For such beliefs, however, the high type will not have an incentive to play  $s = s^*$ .  $\square$

## References

- Cho, In.-Koo and Kreps, David. M., (1987). 'Signaling games and stable equilibria', *Quarterly Journal of Economics*, vol. 102 (May), pp. 179-221.
- Lagerlöf, Johan, (2003). 'Efficiency-enhancing signalling in the Samaritan's dilemma,' *Economic Journal*, forthcoming.